Holy Family Catholic High School

Geometry Review

Version 0.1
Last Modified 2/17/2008
THERE WILL BE NO FORMULAS GIVEN TO YOU DURING THE TEST!

This will be a review of some of the topics needed on the Holy Family placement test. It is by no means complete or a substitute for taking an actual high school level Geometry course. It is intended only for those that already have or are at least most of the way through such a course. At times, it may seem like it is insulting easy but stick with it and I believe that chances are you will learn something. If something is not included in this packet, it may still be included on the test.

First! Here are a few things we always tell our students!-nothing like getting help from a teacher you do not even know yet!

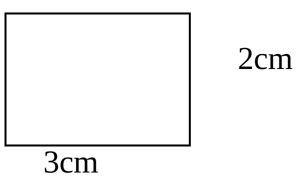
1. Show your work going down! – not across
2. Always show your work!
3. Circle/Box your final answer
4. Reduce all fractions
5. Do not use decimals unless the problem gives you decimals, or it tells you to.
6. Use these steps to solve a problem using a formula
    a. Write out the formula you need
    b. Plug in what you know
    c. Solve for the variable

Area

We are given a polygon and asked to figure out how big it is; when given this flat, two-dimensional shape you are to find the area. We use are when figuring how much carpet we need to buy, or the number of paint gallons needed to cover a wall.

Area of any parallelogram/rectangle/square

When you are given the base and height of parallelogram (I will just say rectangle to save time and paper) you need to use the formula $A = bh$, when $b = base$ and $h = height$. Therefore, for the given rectangle below you would first write $A = bh$ then plug in what you know so…



$A = (3cm)(2cm)$ Then simplify to….

$A = 6cm^2$. NOTE: When you are finding the area of any shape the units will always be squared.

Now, what if they do not give you one of the sides, but give you the total area instead? You follow the same steps as you should for every problem:

1. Write out the formula you need
2. Plug in what you know
3. Solve for the variable
   (Maybe you should keep these in mind for the remainder of your math career)

Back to the problem… you are told that you have a rectangle, with an area of $14cm^2$, a base of $2cm$, and you need to find the height. Well…write out the formula you need

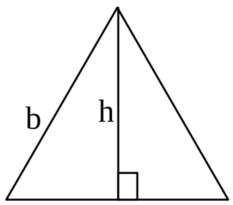
$A = bh$ Plug in what you know…

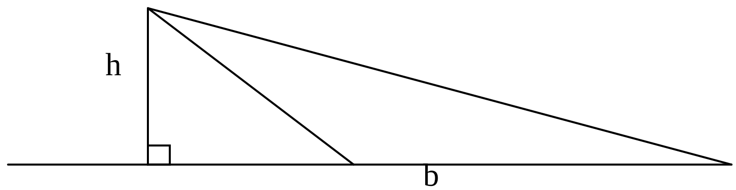
$16cm^2 = (2cm)h$ solve for the variable by dividing both sides by $2cm$ and get your answer

$8cm = h$ so you solved for the height. You would use the same steps if it said solve for the base.

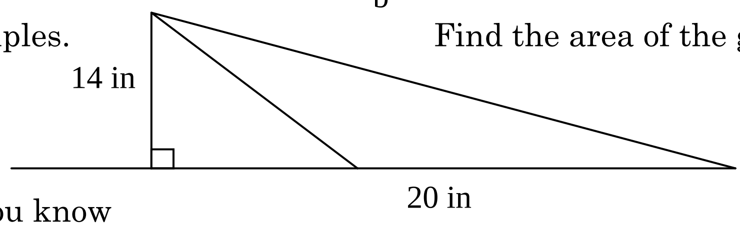
Area of a triangle

First, you need to know what the formula is…. $A = \frac{1}{2}bh$. It is very similar to that of a rectangle, but now you multiply it by ½-which is the same as dividing by 2. In a triangle, the height makes a right angle with the base. Look at the example below, and make sure you understand. Now, the height does not always appear inside the triangle- it can be outside of the triangle as well. This usually happens when the triangle is obtuse. Look at the example below.





So, let us do a couple of examples. Find the area of the given triangle.



First we write out to formula

$A = \frac{1}{2}bh$ then, plug in what you know

$A = \frac{1}{2}(20in)(14in)$ and solve for the variable

$A = 140in^2$

What if you are given the area and need to solve for the base or height??? Same steps as always! You are given a triangle with area of $50\,ft^2$ with a height of $10\,ft$.

$A = \frac{1}{2}bh$ - write down the formula

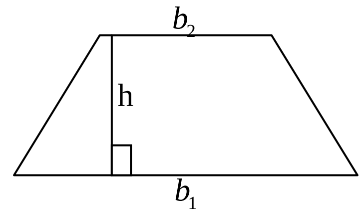
$50\,ft^2 = \frac{1}{2}b(10\,ft)$ - plug in what you know

$50\,ft^2 = b(5\,ft)$ - solve for the variable. First, multiply ½ and 10ft, and then divide both sides by 5 ft.

$10\,ft = b$ - tada! You have your answer!

Area of a Trapezoid

The formula needed is … wait… do you know what a trapezoid is? It is a parallelogram with one pair of parallel sides. Back to the formula, the formula for the area of a trapezoid is: $A=\frac{(b_1+b_2)h}{2}$. Look at the example to make sure you know where all the variables are coming from.



So what happens if the problem looks like this?
You are given a trapezoid with one base of 10 cm, another base of 4 cm, and a height of five cm. Find the area. Follow the steps…

$A=\frac{(b_1+b_2)h}{2}$ - write down the formula

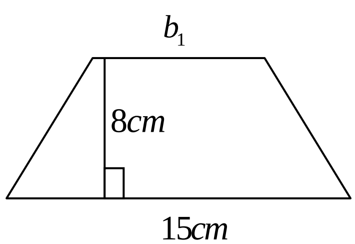
$A=\frac{(10cm+4cm)5cm}{2}$ -plug in what you know

$A=\frac{(14cm)5cm}{2}$ -solve for the variable

$A=\frac{70cm^2}{2}$

$A=35cm^2$ - final answer!

Now, what if it gives you this: Find b_1 when area equals $100cm^2$



$A=\frac{(b_1+b_2)h}{2}$ - write the formula

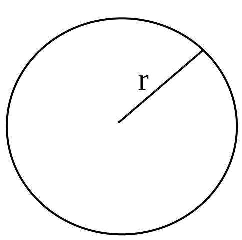
$100cm^2=\frac{(b_1+15cm)8cm}{2}$ - plug in

$100cm^2=(b_1+15cm)4cm$ -reduce 8/2

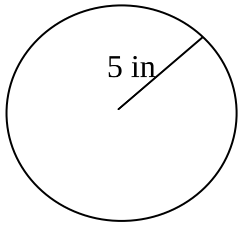
$25cm=b_1+15cm$ - divide both sides by 4

$10cm=b_1$ - Tada!

Area of a circle and circle parts



The formula you need to know for the area of a circle is $A=\pi r^2$. r- represents the radius, which is a segment from the center of the circle to any point on the circle.



Let us try one. This is easier than you think!

$A = \pi r^2$

$A = \pi (5in)^2$

$A = 25\pi in^2$ - final answer!

Here is another one:

You are given a circle with an area of $100\pi\, ft^2$. Find the diameter- remember the diameter is two radii!

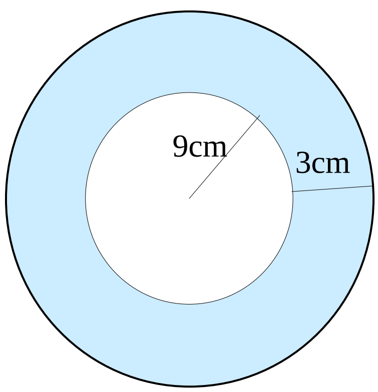
$A = \pi r^2$

$100\pi\, ft^2 = \pi r^2$

$100\, ft^2 = r^2$ - divide by π

$10\, ft = r$ - square root both sides and you are done!

What about something like this, when all you want is the shaded region? Well, find the area of the smaller circle and subtract it from the area of the larger circle.



$A = A_{big} - A_{small}$

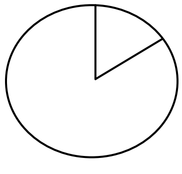
$A = \pi R^2 - \pi r^2$ - Note capital R is radius larger circle

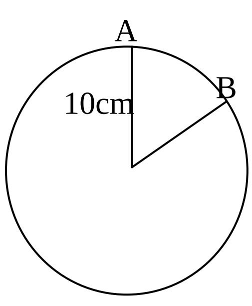
$A = \pi(12cm)^2 - \pi(9cm)^2$

$A = 144\pi cm^2 - 81\pi cm^2$

$A = 63\pi cm^2$ - you can subtract because are like terms!

What about a portion of a circle like a piece of pizza (which is called a sector in math)? Now we are talking about a completely new formula. So if you want to find a portion of the whole circle, you need to find a portion of the area of the whole circle, right? Well if you want half of the area of circle you can multiply it by ½, if you want ¼ you can multiply it by ¼. But what about the pizza without evenly cut slices?





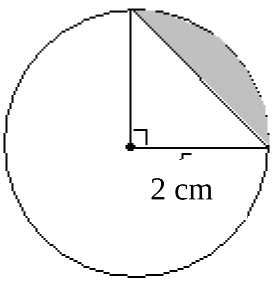
To find the area of a sector you will use the formula $A = \frac{arc^\circ}{360}\left(\pi r^2\right)$. When you have the radius and degree of the arc, you are ready to go. Just follow along. Let us say we want to find the area of the sector in the picture to the left, and we know the arc degree of minor arc AB is 60 degree.

$$A = \frac{arc^\circ}{360}\left(\pi r^2\right)$$

$$A=\frac{60^{\circ}}{360}\left(\pi(10cm)^2\right)$$
$$A=\frac{1}{6}\left(100\pi cm^2\right)$$
$$A=\frac{100\pi cm^2}{6}$$

$A=\dfrac{50\pi cm^2}{3}$ **- Final answer!**

What about a part of a circle, called a segment, like the picture below? Now, you need this formula: $A=\frac{arc^{\circ}}{360}\left(\pi r^2\right)-\frac{1}{2}bh$



Does the last ½ of the formula look familiar? It is the area of the triangle. What you take is the area of a sector and subtract that area of a triangle. Now do you see where the formula came from?

Let us use the picture above to do an example: $A=\frac{arc^{\circ}}{360}\left(\pi r^2\right)-\frac{1}{2}bh$

$A=\frac{90}{360}\left(\pi(2cm)^2\right)-\frac{1}{2}(2cm)(2cm)$ -the radius, base, and height are all the same because it is a right triangle. The arc degree is 90 because the intercepted arc is equal to the central angle.

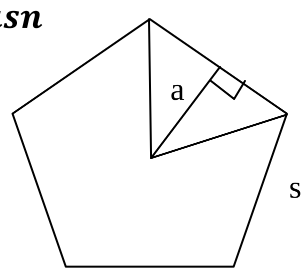
$A=\frac{1}{4}\left(4\pi cm^2\right)-\frac{1}{2}(4cm^2)$

$A=1\pi cm^2-2cm^2$ - Final answer Because they are not like term, so you cannot add them to make -1cm squared.

Area of a regular polygon (a polygon that is equilateral and equiangular)

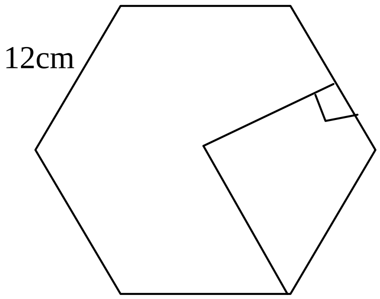
$\boldsymbol{A=\frac{1}{2}asn}$

a = apothem, s = side length, and n = number of sides



For this picture, n would equal five because it is a pentagon.

Let us do an example: You are given a regular hexagon and need to find its area.

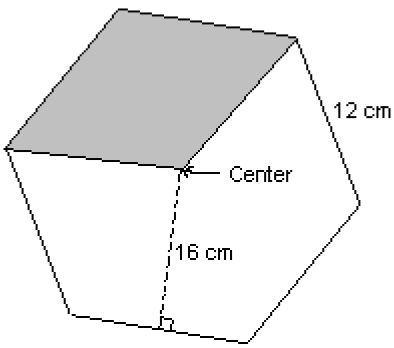


$A = \frac{1}{2}asn$

$A = \frac{1}{2}(9cm)(12cm)6$

$A = 324cm^2$

What happens if you only want to find that area of part of a regular polygon? Well, then you only use the number of sides that are included. For example:



Find the area of the shaded region of the regular hexagon.

$A = \frac{1}{2}asn$

$A = \frac{1}{2}(16cm)(12cm)2$ - n = 2 because only two sides are included in the shaded region

$A = \frac{1}{2}384cm^2$

$A = 192cm^2$ - final answer

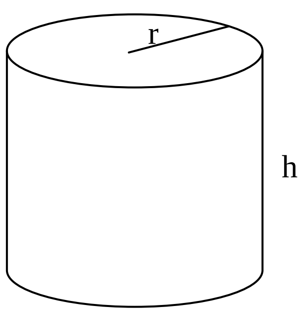
Volume and Surface Area

Volume is measurement of the amount of space contained in a solid. When we talk about volume, we are talking about 3D shapes. The other thing to remember is that when you talk about volume the units will always be cubed. For example cm^3. When we talk about Surface Area we mean the sum of the areas of all the faces or surfaces that enclose the solid.

Volume of a Prism and Cylinder

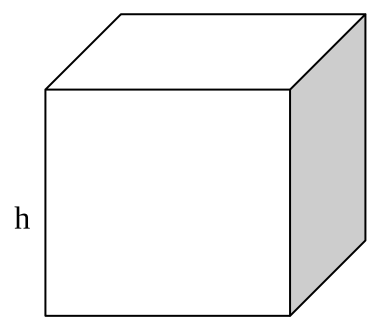
You can use the same formula for both of these shapes! This is good! The formula is $V = Bh$.
Now, the hard part about this formula is that B stands for area of the base, so you need to know the shape of the base is and the formula to find the area of the shape. It is more difficult than it looks.

Cylinder



$V = \pi r^2 h$

Prism



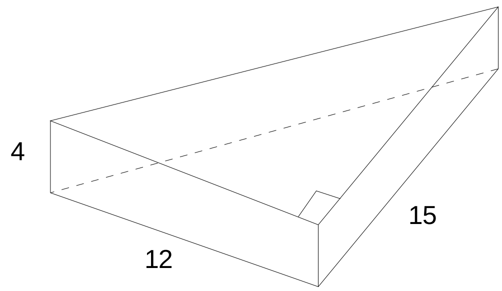
$V = (bh)h$

Now, in the prism each h represents a different number. The h inside the parentheses represents height of the base, and the h outside of the parentheses represents the height of the entire prism. Confusing sometimes, I know.

Let's do an example. Find the volume of a cylinder with a height of 13 cm and diameter of 6cm.

$V = \pi r^2 h$
$V = \pi(3cm)^2(13cm)$ - remember the radius is ½ the length of the diameter.
$V = \pi(9cm^2)(13cm)$
$V = 117\pi cm^3$ - Final answer!

Here is another example. Find the volume of the given prism. All units are in inches.



$V = (\frac{1}{2}bh)h$

$V = \left(\frac{1}{2}(12cm)(15cm)\right)4cm$

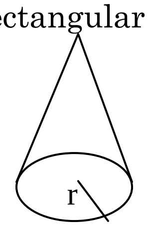
$V = (90cm^2)4cm$

$V = 360cm^3$ - Final answer

Volume of a Cone and Pyramid

The volume of a cone and pyramid is very similar to the formula to find the volume of a cylinder or prism. All you need to do is put one more part into the formula. $V = \frac{1}{3}Bh$ This means that if you have a prism and a pyramid with the same size base, it will take 3 pyramids to fill one prism.

Cone



Pyramid-there are three of them here. Each with a rectangular base

Here are a few examples.

1. Find the volume of a cone with a diameter of 18 in. and a height of 9 in.

So we know that h = 9 in and r = 9 in

$V = \frac{1}{3}\pi r^2 h$

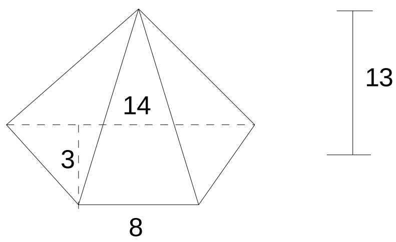
$V = \frac{1}{3}\pi(9in)^2(9in)$

$V = \frac{1}{3}\pi 81in^2(9in)$

$V = \frac{1}{3}\pi 729in^3$

$V = 243\pi in^3$

2. Make sure you remember to formula to find the area of a trapezoid. All units are in centimeters

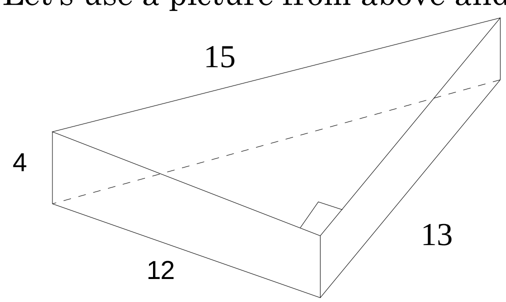


$V = \frac{1}{3}(\frac{(b_1+b_2)h}{2})$h
$V = \frac{1}{3}\left(\frac{(14cm+8cm)3cm}{2}\right)13cm$
$V = \frac{1}{3}\left(\frac{66cm^2}{2}\right)13cm$
$V = \frac{1}{3}(33cm^2)13cm$
$V = (11cm^2)13cm$
$V = 143cm^3$

Surface Area(S) of a Polyhedron

This is easier than you think. Whenever finding the surface area of a polyhedron, just find the sum of faces areas. Translation: find the area of each side and then add them together, including the base(s)!

Let's use a picture from above and add one number to it, to make it work.



To do this problem we need to find the area of each side. So, in the end we find the area of five different shapes; but are the two triangles the same? Hmm… does that help us? Let's get started.

S $=2\frac{1}{2}bh + bh + bh + bh$ is each bh different? Yep
S $= (12in)(13in) + (4in)(15in) + (4in)(12) + (4in)(13in)$
S$= 156in^2 + 60in^2 + 48in^2 + 52in^2$
S $= 316in^2$- Final Answer!

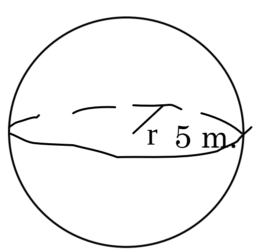
Does it matter how many sides the polyhedron has? No. With every surface area problem, find the area of each side and then add them all together.

Volume of a Sphere

To find the volume of a sphere you must use the formula $V = \frac{4}{3}\pi r^3$. Does it look wrong with the

improper fraction $\frac{4}{3}$? I know, I understand your confusion,

but



it is right. So, let's say the radius of the circle to the left is

$V = \frac{4}{3}\pi r^3$
$V = \frac{4}{3}\pi(5m)^3$
$V = \frac{4}{3}\pi(125m^3)$
$V = \frac{500\pi}{3}m^3$ Final Answer!

What if you wanted to find the Volume of a hemisphere(1/2 of a sphere)? Well, just divide the entire volume in 2. And then you will have your answer.

Surface Area of a Sphere

To find the surface area of a sphere is a little different then finding the surface area of a polyhedron, because there are no sides to a sphere. So, we are stuck just using one formula for the surface area of a sphere. $S = 4\pi r^2$

We will use the sphere from the problem above and now find the surface area of it.
$S = 4\pi r^2$
$S = 4\pi(5m)^2$
$S = 4\pi 25m^2$
$S = 100\pi m^2$ – Final Answer!

Now, what about a hemisphere? If all we do it divide the surface area is half, we will miss the base of the surface area. What shape is the base of a hemisphere? Circle? That is right!

So the formula for the surface area of a hemisphere will be $S = 2\pi r^2 + \pi r^2$. It is $2\pi r^2$ instead of 4, because we are only using half of the surface area of the sphere.

Here is an example:
Find the surface area of a hemisphere with a radius of 10 ft.

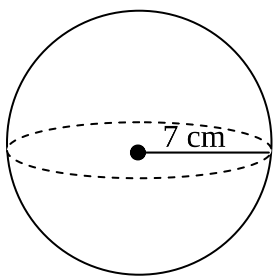
$S = 2\pi r^2 + \pi r^2$
$S = 2\pi(10ft)^2 + \pi(10ft)^2$
$S = 2\pi 100ft^2 + 100\pi ft^2$
$S = 200\pi ft^2 + 100\pi ft^2$
$S = 300\pi ft^2$ –Final Answer!

Practice Problems

1. Find the volume and surface area of a sphere with a radius of 9 in.

Volume = ____________________ Surface Area = ________________________

2. Find the surface area of the sphere.

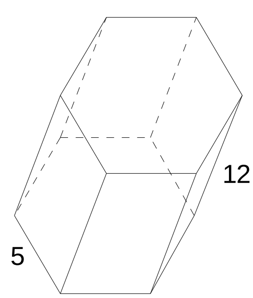


Surface Area = ______________________

3. The surface area of a sphere is 36π cm^2. What is its volume?

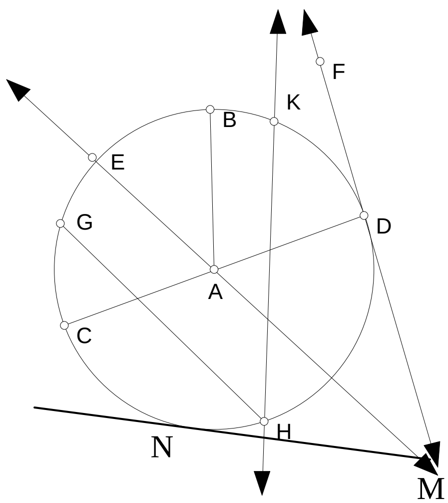
Volume =_____________________

4. This is a regular hexagonal prism. Each side is length 5 in. The apothem a = 4 in and the height is 12 in.



Circles

We all know what a circle is, but do we all know about the special properties a circle has. Here, we will go through many definitions for the parts of the circle and then explain the special characteristics that each part has.



To the left, you will find a circle with many things added to it. First the vocabulary!

Secant- a line intersecting a circle twice. EX. $\overleftrightarrow{HK}$

Tangent –A line/segment/ray intersecting a circle once. EX. $\overleftrightarrow{DF}$ and $\overline{MN}$

Point of Tangency- point at which the tangent line intersects the circle. EX. D

Chord- A segment intersecting the circle twice. EX $\overline{GH}$

Diameter- A chord that passes through the center. EX $\overline{CD}$

Radius- segment from the center of a circle to a point on the circle. EX. $\overline{EA}, \overline{BA}, \overline{AD}, \overline{AC}$

Central Angle- An angle in the circle with it's vertex at the center of the circle. EX $\angle CAE, or \angle EAB, or \angle BAD$

Inscribed Angle- An angle in the circle with it's vertex on the circle. EX. $\angle GHK$

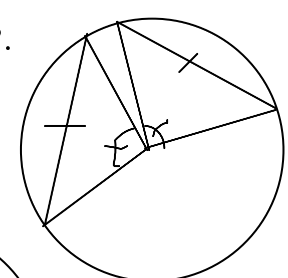
Now, remember the picture from above. We will be using it when going through special properties.

SPECIAL PROPERTIES FOR A TANGENT:

1. A tangent to a circle is perpendicular to the radius drawn to the point of tangengcy. So, $\overline{CD} \perp \overleftrightarrow{DF}, and \angle ADF \;\; is \;\; 90^\circ$.
2. Tangent segments to a circle from a point outside the circle are congruent. So, $\overline{MN} \cong \overline{MD}$.

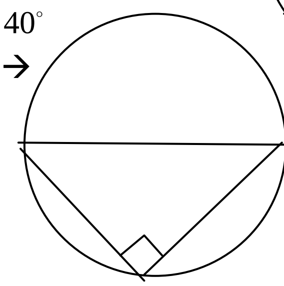
SPECIAL PROPERTIES FOR A CENTRAL ANGLE:

1. A central angle is congruent to it's intercepted arc. So, $\angle EAB \cong \widehat{EB}$.
2. Congruent cords determine congruent central angles. Look →

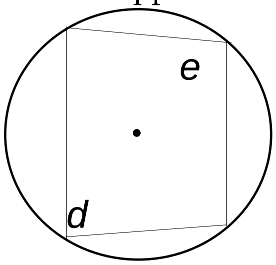


SPECIAL PROPERTIES FOR AN INSCRIBED ANGLE:

1. The measure of an inscribed angle is ½ the measure of its intercepted arc. EX. $if \;\; arcGK = 80^\circ$ then $m\angle GHK = 40^\circ$
2. Angles inscribed in a semicircle are right angles. →
3. Inscribed angles that intercept the same arc are congruent.



4. The opposite angles in a quadrilateral inscribed in a circle are supplementary. → $d + e = 180$ degrees



Circumfrence

The circumference of a circle is the distance around the circle, kind of like the perimeter when talking about a polygon. Ratio of the circumference and the diameter of a circle is always the same, $3\,{}^{1}\!/_{7}$. Instead of always using an irrational number mathematicians made up the symbol and name pi, π. So, to find the circumference of the circle all you need to do is use the formula $C = \pi d$, or since the diameter is equal to 2 radii $C = 2\pi r$

EXAMPLE:

Given a circle with the radius of 4 ft, find the circumference.

$C = 2\pi r$

$C = 2\pi(4\ ft)$

$C = 8\pi\ ft$ → Final answer

Arc Length

Arc Length is different than arc measure. Measure has units of degrees, length has units in in. or cm. or ft or so on. Finding the length would be when you would use a ruler to measure the distance from one point on the circle to another point on the circle.

Here is the formula you will need to know Arc Length = $\frac{arc^\circ}{360}(circumfrence)$. For circumference you need to decide which formula to use, depending on what you are given or what you need to solve for.

EXAMPLE:

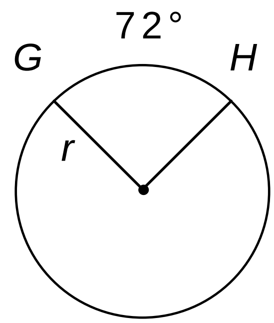
What is the length of the arc that measures 80° on a circle with radius of 5 m?

Arc Length $= \frac{arc^\circ}{360}(2\pi r)$

Arc Length $= \frac{80^\circ}{360}(2\pi 5m)$

Arc Length $= \frac{2}{9}(10\pi m)$

Arc Length $= \frac{20\pi}{9} m$ →final answer.



EXAMPLE 2: If the arc length of $\widehat{GH} = 8\pi$ cm. Find r. →

$$\text{Arc Length} = \frac{arc^{\circ}}{360}(2\pi r)$$

$$8\pi cm = \frac{72}{360}(2\pi r)$$

$$8\pi cm = \frac{1}{5}(2\pi r)$$

→ multiply both sides by 5 to get rid of 1/5

$$40\pi cm = 2\pi r$$

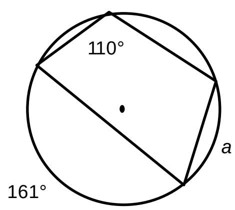
→divide both sides by 2pi

$$20cm = r$$

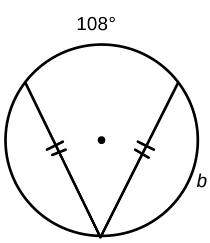
final answer!

Practice Problems

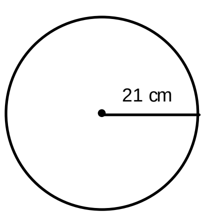
1. . a =



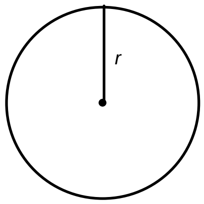
2. b =



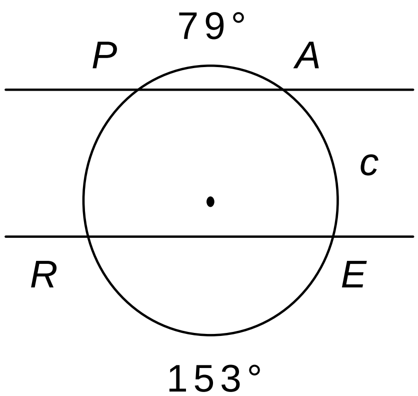
3. What is the circumference?



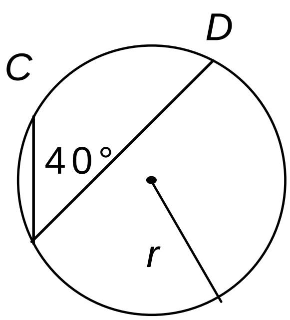
4. Circumference is 24π m. r =



5. $\overleftrightarrow{PA} \parallel \overleftrightarrow{RE}$ c = __________



6. r = 18 cm, arc length of $\overset{\frown}{CD}$ = __________



7. If the diameter of the moon is 3475 km and an orbiting lunar station is circling 21 km above the lunar surface, find the distance traveled by the lunar station in one orbit.

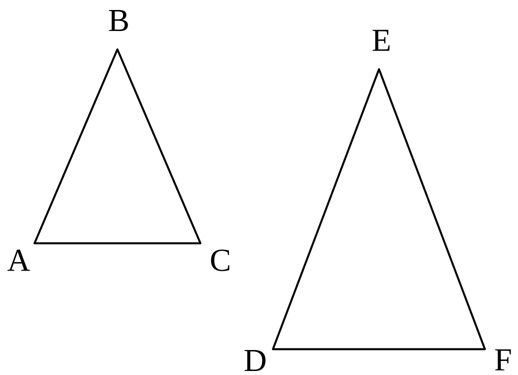
8. A circle has $\overset{\frown}{ABC}$ whose measure is 80° and length is 88π. What is the diameter of the circle?

Similarity

Similar Polygons

Two polygons are considered similar when they have congruent corresponding angles and proportional corresponding sides. The symbol for similar is ~. For example, if I stated that HOLY~FAMI the $\angle H \cong \angle F, \angle O \cong \angle A, \angle L \cong \angle M$, and $\angle Y \cong \angle I$ must be true, along with $\frac{HO}{FA} = \frac{OL}{AM} = \frac{LY}{MI} = \frac{YH}{IF}$. If just one of these 8 things is not true then the two polygons are not similar.

To determine if two polygons are similar you must check all of the corresponding angles and sides to make sure they work. Follow the example below:



Is $\Delta ABC \sim \Delta DEF$? To answer, we must show that $\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$ and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$. First, take your protractor and measure each angle, are the corresponding angles congruent? Yes! You are right! we are half way there. Now, measure each side with a ruler, centimeters would be easier to work with, the set up the proportions. Do they all equal each other? YES! You are right! So $\Delta ABC \sim \Delta DEF$ is true!

Here are few conjectures to help you while working with similar triangles:

SSS Similartity Conjecture If the three sides of one triangle are proportional to the three sides of another triangle, then the two triangles are similar. So, no need to do all 6 checks!

AA Similarity Conjecture If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.

SAS Similarity Conjecture If two sides of one triangle are proportional to two sides of another triangle and the included angles are congruent, then the two triangles are similar.

Proportions with Area and Volume

If two polygons or circles have lengths of corresponding sides or radii in the ratio of m/n then their areas are in the ratio of m^2/n^2. What does this mean? Well, if you have two similar polygons and the ratio of their sides is $3/5$, then the ratio of their areas is $9/25$. If you know the area of the smaller polygon equals $12cm^2$, you can then find the area of the larger polygon(x) by setting up a ratio.

$\frac{9}{25} = \frac{12cm^2}{x}$ → To solve all we need to do is cross multiply.

$9x = 300cm^2$ →Next, divided both sides by 9.

$x = \frac{100}{3} cm^2$ → So, the area of the larger polygon.

If two similar solids are corresponding dimensions in the ratio of m/n, then their volumes are in the ratio of m^3/n^3. Do you see a pattern? Ratios for area are squared, and ratios for volume are cubed… hmm… just like their units!

Example: Two similar cylinders' radii have a ratio of 2/5. The smaller cylinder has a volume of 198 ft^3. What is the volume of the larger cylinder?

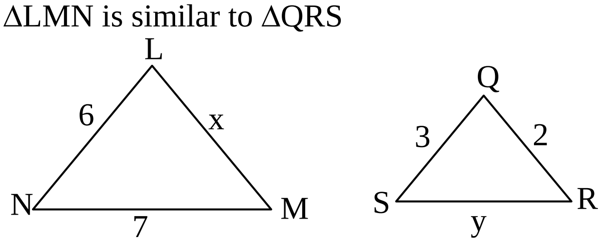
$$\frac{8}{125} = \frac{198\,ft^3}{x}$$
$$8x = 24,750\,ft^3$$
$$x = \frac{12,875}{4}\,ft^3$$

Practice Problems

1. $\frac{10}{17} = \frac{x}{68}$ x = ________

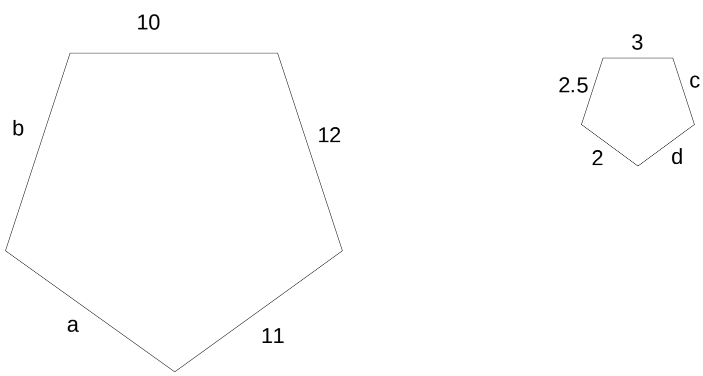
2. $\frac{y+3}{12} = \frac{y}{7}$ y = ________

3. ΔLMN is similar to ΔQRS x = _________

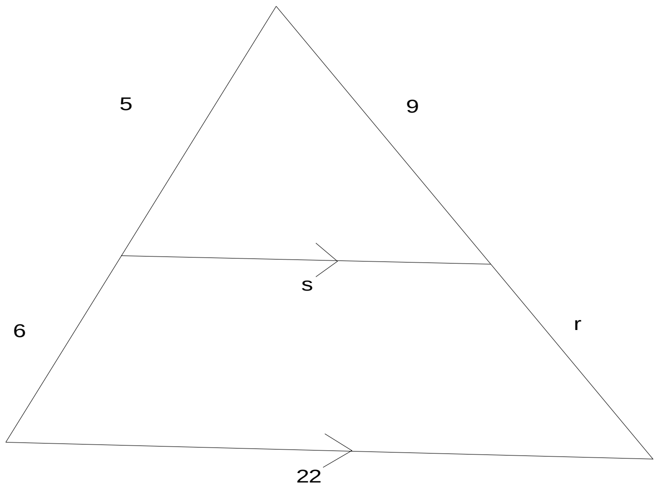


4.

a = ___________ c = _______________



5. r = __________ s = _________

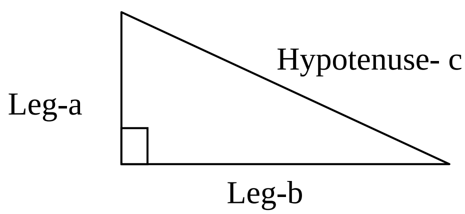


6. If a 36-foot tree casts a 28-foot shadow at the same time that a nearby building casts a 70-foot shadow, how tall is the building. Draw a diagram for this problem!
Height = _________

7. You have 2 right cylinders that are similar. The volume of the larger cylinder = $4608\pi cm^3$. The radius of the small cylinder if 9 cm and the height is 24cm. What is the height (H) of the larger cylinder? (Draw a picture first!)

Right Triangles

A long time ago this person name Pythagoras loved math and discovering new concepts concerning math. He really enjoyed the right triangle. He discovered something so important about the right triangle they named the theorem after him. Before we are carried away with Pythagoras we first have to make sure we all understand what a right triangle is. Here goes…



A right triangle is a triangle with one right angle. Meaning one angle measures 90 degrees. The sides that form the right angle are called legs(a and b) and the side opposite the right angle and the longest side is called the hypotenuse(c).

Pythagorean Theorem

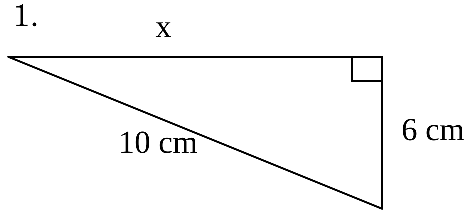
We use the Pythagorean Theorem a lot! Ever wonder how they measure TVs? It is by measuring the diagonal on the screen; that diagonal is also the hypotenuse of a right triangle. You will use the Pythagorean Theorem when you want to find one side of a RIGHT triangle when given the other two sides.

I will not make you wait any longer… here is the theorem you have been waiting for.

$a^2 + b^2 = c^2$ Pythagorean Theorem. Remember that a, b, c, come from the sides of the triangle. Also, remember that c is ALWAYS the hypotenuse! a and b can switch around, but c is always the hypotenuse.

Here are some examples:

1.



Solve for x. First… what letter is x representing, a, b, or c? b, you are right.

$a^2 + b^2 = c^2$ - you cannot take the √ of everything right away. It does not work!

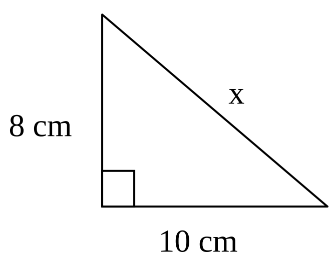
$(6cm)^2 + x^2 = (10cm)^2$

$36cm^2 + x^2 = 100cm^2$ - subtract 36cmsq. Both sides

$x^2 = 64cm^2$

x = 8cm

2. Solve for x.



$a^2 + b^2 = c^2$

$(8cm)^2 + (10cm)^2 = x^2$

$64cm^2 + 100cm^2 = x^2$

$164cm^2 = x^2$

$\sqrt{164cm^2} = \sqrt{x^2}$ - now reduce the radicals

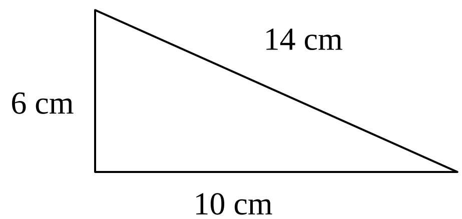
$2\sqrt{41}cm = x$ - Final Answer!

Converse Pythagorean Theorem

You may get a problem that says, "is the given triangle a right triangle?" To answer this question you must plug the three given sides into the Pythagorean Theorem. What numbers go where you ask. Well, the longest side will be c and the other two sides will be a and b. If the statement is true, then it is a right triangle. If the statement is false, it is not a right triangle.

Like so:

1. Determine if the given triangle is a right triangle or not. It looks like a right triangle, but is it?

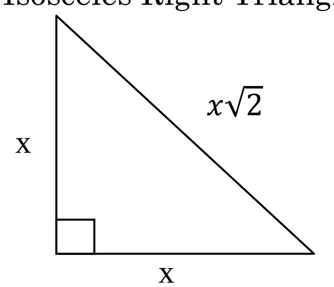


$a^2 + b^2 = c^2$
$6^2 + 10^2 = 14^2$
$36 + 100 = 196$
$136 \neq 196$ the statement is not true, so it is not a right triangle

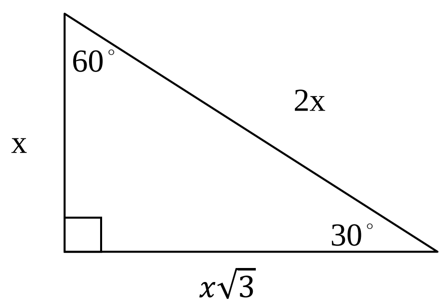
Special Right Triangles

Within the world of right triangles there are two special right triangles the 30-60-Right Triangle and the 45-45 Right Triangle also known as the isosceles right triangle. With these two triangles, their sides always follow a pattern.

Isosceles Right Triangle



30-60 Right Triangle



Right Triangle Trigonometry

Trigonometric Ratios

Have you ever wondered how to find the height of an extremely tall building? If you have, you are in for a surprise – it's really simple! When early mathematicians and astronomers calculated the ratios of the sides for different triangles, they discovered a pattern. Mathematicians from a variety of early civilizations, such as Babylonian, Egyptian, Greek, Hindu, and Arabic, recorded the ratios for different angles and recorded them in tables. These tables were used to calculate the height of objects or long distances. The study of the relationships between the angles and the sides of right triangles is called – you guessed it – right triangle trigonometry.

Thankfully, we no longer use these tables in modern day mathematics. The ratios recorded on these tables can be supported by the ***AA similarity conjecture***. Let's refresh our memory:

If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

We can also say that if two triangles are similar, then the ratio of two sides in one triangle is equal to the ratio of the corresponding sides in other similar triangles (***Polygon Similarity Postulate***). So we can take what we know about the ratio of the sides of a triangle with a particular acute angle and apply that to a similar triangle with the same acute angle.

Before we go further, I need to pause to tell you a little about how we represent angles in trigonometry. As you know, variables (or unknowns) are typically indicated with a letter such as x. Angles are also represented by letters but usually from the Greek alphabet. Normally angles are named θ (theta). Sometimes, especially when talking several different angles at a time, the letters α (alpha) or β (beta) are used. Other letters from the Greek and English alphabet are used for angles as well.

Let's take a look at a right triangle and name the different parts. As you know, the longest side of a triangle is called the **hypotenuse** and is across from the angle with the greatest measure (***Side-Angle Inequality Conjecture***). In the case of a right triangle, the right angle is the largest angle so the hypotenuse is across from that. The leg that is opposite of the angle, which in this case is θ, is the opposite side. The leg that is adjacent to the angle θ is the adjacent side. As you can see in Figures 1 & 2, the angle referred to does dictate which leg is opposite and which one is adjacent, as the definitions of the words would suggest.

Figure 1

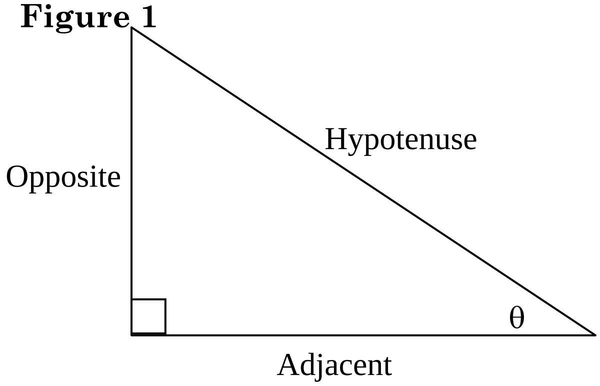
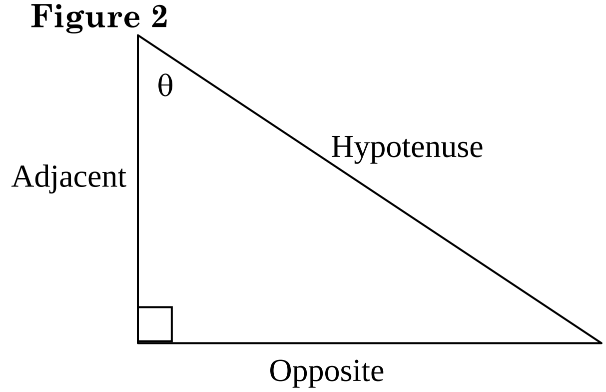


Figure 2



There are six ratios formed by pairs of sides of a triangle. In Geometry, you need to know three of these – sine, cosine, and tangent. When you are in Algebra II, you will learn the other three. To help you remember them, you should memorize this mnemonic SOH-CAH-TOA.

sin θ = $\frac{opposite}{hypotenuse}$ cos θ = $\frac{adjacent}{hypotenuse}$ tan θ = $\frac{opposite}{adjacent}$

These mean…

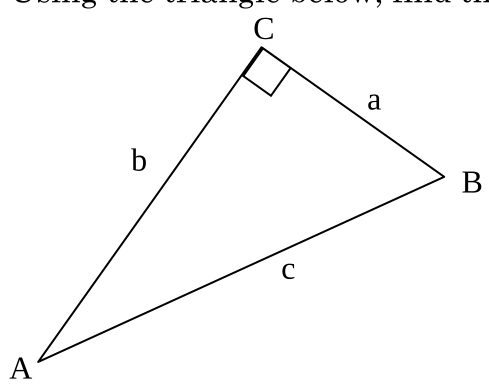
Sine of θ = $\frac{length\ of\ the\ opposite\ side}{length\ of\ the\ hypotenuse}$

Cosine of θ =$\frac{length\ of\ the\ adjacent\ side}{length\ of\ the\ hypotenuse}$

Tangent of θ =$\frac{length\ of\ the\ opposite\ side}{length\ of\ the\ adjacent\ side}$

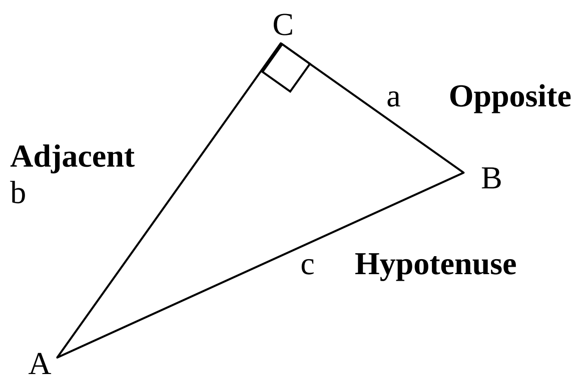
Example

Using the triangle below, find the three trigonometric ratios for angle A.



Example Solution

First of all, you should note that a right triangle is a triangle with one angle that measures ninety degrees. The right angle does not have to be in the lower left-hand corner, although it is frequently drawn that way. Let's start by labeling the sides of the triangle in terms of angle A.



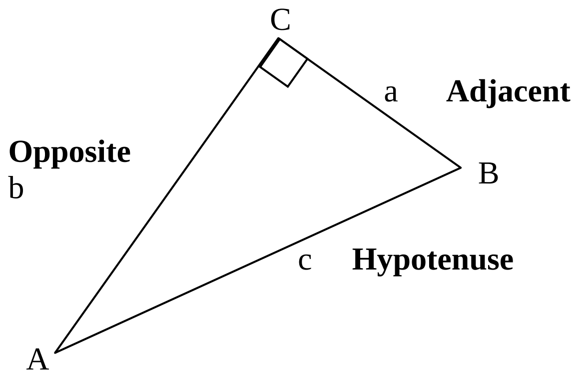
Now, write down what you know about sine, cosine, and tangent and fill in the information from the diagram.

sin A = $\frac{opposite}{hypotenuse} = \frac{a}{c}$

cos A = $\frac{adjacent}{hypotenuse} = \frac{b}{c}$

tan A = $\frac{opposite}{adjacent} = \frac{a}{b}$

Just for fun…what are the trigonometric ratios for angle B? Since the angle changed, so do the sides with the exception of the hypotenuse.



So,

sin B = $\frac{opposite}{hypotenuse} = \frac{b}{c}$ cos B = $\frac{adjacent}{hypotenuse} = \frac{a}{c}$ tan B = $\frac{opposite}{adjacent} = \frac{b}{a}$

This process can be used regardless whether the lengths of the sides are labeled with numbers or variables.

Evaluating Trigonometric Functions Using Calculators

Remember those tables I mentioned earlier? Lucky for you, you don't have to keep one of those in your pocket – you have a calculator! Take a look at your scientific calculator. Notice the buttons labeled sin, cos and tan? That's right, just push that button and you get your answer. First of all, you need to recall your elementary years in mathematics. Remember learning about fractions? Fractions are representative of dividing. The fraction ½ is the same as 1 divided by 2. You then spent days toiling over long division problems until your teacher finally gave in and let you use your calculator to find the decimals. My point is, fractions represent division which also have decimal solutions. When you use your calculator to find the trigonometric values of angles, it will give you a decimal solution. Some calculators will convert decimals to fractions, if it can determine its value which isn't always possible with trigonometric functions. Depending on your particular brand of calculator, the process that you find the trigonometric value of an angle may be different. Most calculators will have you push the appropriate button, for instance sin if you're looking for the sine ratio, followed by the value of the angle. Some calculators make you enter the value of the angle and then push the appropriate button. Make sure you know how your calculator works. If you're not sure, you can look at your owner's manual or just try both ways and see which one works (using a known trigonometric value of course).

Example

Using your calculator, find the sin 15^o. Round your answer to the nearest one thousandth.

Solution

The process on my calculator is push the sin button followed by 1 and 5 then =.

The result rounded to the nearest one thousandth is 0.259. If your calculator says 0.650, then you need to change the mode of your calculator. Please see your calculator manual or your teacher for assistance.

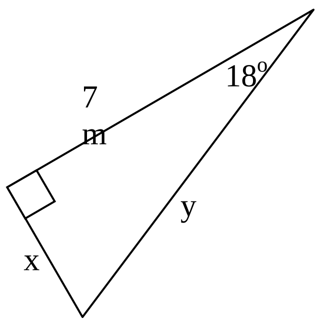
Here are a few more:
cos 38 = 0.788
tan 20 = 0.364

Using Trigonometric Ratios

You can use what you've learned so far to find the side of any triangle. I will also be a good time to remind you of the ***Pythagorean Theorem***:
In a right triangle, if a and b are the lengths of the legs and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$.
Let's try a few different problems.

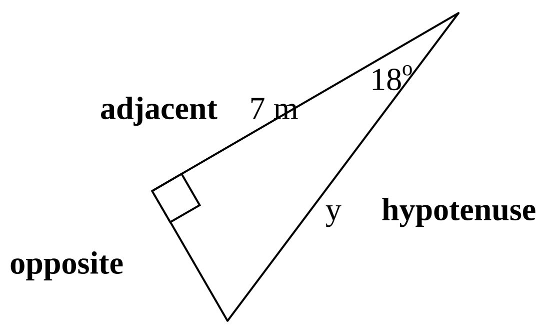
Example

Use your calculator to approximate the length of the missing sides. Express your answer accurate to the nearest one tenth.



Example Solution

Let's start with filling in a trigonometric ratio. We need to decide which one would be most useful so let's take a look at the triangle. Using the angle as a reference point, we'll identify the opposite, adjacent, and hypotenuse just like we did when finding the trigonometric ratios.



We know the measurements of the angle and the adjacent side. If you look at the trigonometric ratios you will notice that two of them involve the angle and adjacent side – cosine and tangent. Now you just need to decide if you want to find the hypotenuse or the opposite first. Let's solve for the hypotenuse. So if we're using the angle, adjacent side and the hypotenuse we will use cosine. Fill in the cosine ratio.

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

$\cos 18^{o} = \frac{7}{y}$

If we multiply both sides of the equation by y we will have

y cos 18^{o} = 7

Then divide both sides by cos 18^{o} to get

$$y = \frac{7}{\cos 18^{o}}$$

Using your calculator you can determine that y = 7.4 m. Don't forget to label your answers!!

Don't forget that the question asked you to find the length of the side to the nearest one tenth.

We also need to find x. To do this we could use the sine or tangent ratios or the Pythagorean Theorem. The quickest way would be to use the Pythagorean Theorem.

Substitute the lengths of the sides into $a^2 + b^2 = c^2$

$7^2 + x^2 = 7.4^2$

Subtract 7^2 from both sides.

$x^2 = 5.76$

Square root both sides

$x = \pm 2.4$

Since we are looking for the side of a triangle, the only possible solution would be the positive root so x = 2.4 m.

Solution x = 2.4 m and y = 7.4 m

Inverse Trigonometric Functions

You may have noticed the $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$ that are the 2nd function of your sin, cos, and tan buttons. You may have also wondered, if I know the ratio of two sides of a triangle, how do I find the angle?

Inverse trigonometric functions, such as $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$, are used to find the angle measurements of right triangles. You may have already studied inverse functions in your Algebra class, but many first year Algebra courses do not cover this material. A **function** is a relation for which each x value has only one y value. A **relation** is a set of ordered pairs which can be represented by an equation, an x-y table, a mapping, or graph. An **inverse function** is obtained by switching the input and output of a function. Typically, this means that you simply switch the x and y values or variables. In other words, you use the trigonometric ratio (output) to determine the angle (input).

Example

Evaluate $sin^{-1}\left(\frac{3}{4}\right)$. Round your answer to the nearest degree.

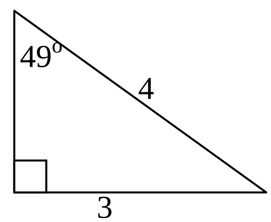
Example Solution

On your calculator, push the following buttons 2nd sin (3 ÷ 4) =

Your answer is 49^{o} remembering that the directions said to round to the nearest degree.

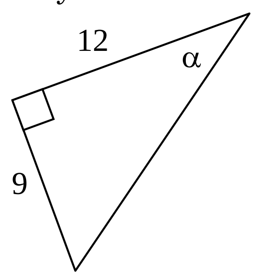
What does this mean?

Let's take a look at the triangle that it represents. If we rewrite $sin^{-1}\left(\frac{3}{4}\right) =$ 49^o, we would get sin $49^o = \frac{3}{4}$ So the angle is 49^o, the opposite side is 3 and the hypotenuse is 4.



Example

Find the missing angle α. Round your answer to the nearest degree.



Example Solution

Which sides of the triangle do we know? That's right, we know that the adjacent side is 12 and the opposite side is 9. We also know that the tangent trigonometric function relates those two sides. Let's fill in the tangent trigonometric function with what we know.

tan θ = $\frac{opposite}{adjacent}$

tan α = $\frac{9}{12}$

We're looking for an angle measurement and we know that the inverse trigonometric functions are used to find angle measurements. If we rewrite the tangent ratio, solving for the angle we would have.

α = $tan^{-1}\frac{9}{12}$

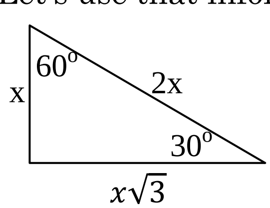
Using our calculator, rounding to the nearest degree, we will get

α =37^o

Special Triangles

When you were learning about right triangles, you learned about two special ones: 30-60-90 and 45-45-90 (Isosceles Right Triangle). Let's recap what you know about these special triangles.

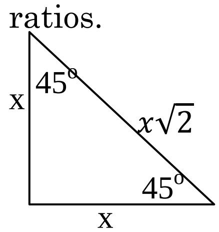
In a 30-60 right triangle, if the side opposite the 30^o angle has length x, then the hypotenuse has length 2x and the longer leg, the side opposite the 60^o angle, has length $x\sqrt{3}$. Let's use that information to learn some special trigonometric ratios.



sin 30° = $\frac{x}{2x}$ cos 30° = $\frac{x\sqrt{3}}{2x}$ tan 30° = $\frac{x}{x\sqrt{3}}$ sin 60° = $\frac{x\sqrt{3}}{2x}$ cos 30° = $\frac{x}{2x}$ tan 30° = $\frac{x\sqrt{3}}{x}$

sin 30° = $\frac{1}{2}$ cos 30° = $\frac{\sqrt{3}}{2}$ tan 30° = $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ sin 60° = $\frac{\sqrt{3}}{2}$ cos 30° = $\frac{1}{2}$ tan 30° = $\sqrt{3}$

In an isosceles right triangle, if the legs have length x, then the hypotenuse has length $x\sqrt{2}$ (***Isosceles Right Triangle Conjecture***). We also know, using the definitions of isosceles triangle and right triangle, the angle measurements would be 45°, 45°, and 90°. Let's use that information to learn some more special trigonometric ratios.



sin 45° = $\frac{x}{x\sqrt{2}}$ cos 45° = $\frac{x}{x\sqrt{2}}$ tan 45° = $\frac{x}{x}$

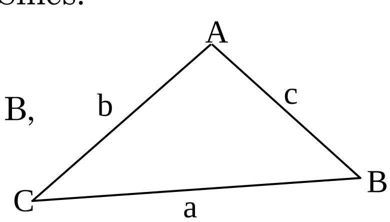
sin 45° = $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ cos 45° = $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ tan 45° = 1

When you take Algebra II or Honors Algebra II, you will spend more time working with these and you should know them from memory.

The Law of Sines

The trigonometric ratios we've talked about so far relate to only right triangles. However, you can find the lengths of sides and angle measurement for all triangles using trigonometry as well. To do this, we use the Law of Sines.

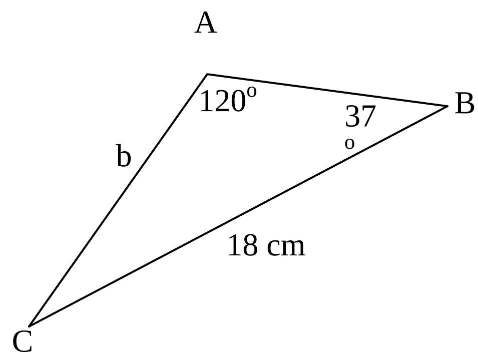
For an acute triangle with angles of measure A, B, and C and sides of lengths a, b, and c (a opposite A, b opposite B, and c opposite C), $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ (***Law of Sines***).



The law of sines can be used to find the measures of sides and angles in a triangle if you are given two angles and one side or two sides and an angle that is opposite one of the sides.

Example

Find side b for the triangle shown below. Round your answer to 3 decimal places.



Example Solution

Start by filling in the law of sines.

$$\frac{\sin 120^o}{18} = \frac{\sin 37^o}{b} = \frac{\sin C}{c}$$

Since we don't know enough information about angle C and side c, we don't need to use that ratio leaving us with

$$\frac{\sin 120^o}{18} = \frac{\sin 37^o}{b}$$

Using our Algebra skills, we know that we can cross multiply ratios.

b sin 120^o = 18 sin 37^o

We want to know what b is so divide both sides by sin 120^o.

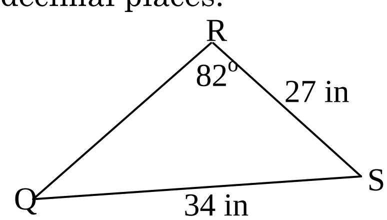
$$b = \frac{18 \sin 37^o}{\sin 120^o}$$

Using our calculator to solve, remembering to round to 3 decimal places, we get

b = 12.508 cm

Example

Find the measurement for angle Q for the triangle shown below. Round your answer to 3 decimal places.



Example Solution

Start by filling in the law of sines.

$$\frac{\sin 82^o}{34} = \frac{\sin S}{s} = \frac{\sin Q}{27}$$

Since we don't know enough information about angle S and side s, we will not use that information.

$$\frac{\sin 82^o}{34} = \frac{\sin Q}{27}$$

Cross multiple

27 sin 82^o = 34 sin Q

Divide both sides by 34

$$\frac{27 \sin 82^o}{34} = \sin Q$$

Since we're looking for an angle, we know we need to find the inverse

Q = $\sin^{-1} \frac{27 \sin 82^o}{34}$

Q = 51.849^o (Directions said to round to 3 decimal places)

When you take Algebra II or Honors Algebra II, you will learn about ambiguous cases for triangles. For now, this is what you need to know about the law of sines.

The Law of Cosines

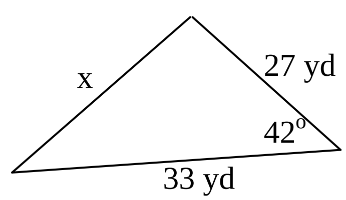
How do you solve triangles that don't give you two angles and one side or two sides and an angle that is opposite one of the sides? Don't worry, mathematicians have an answer for almost everything!

For any triangle with sides of lengths a, b, and c, and with C the acute angle opposite the side with length c, $c^2 = a^2 + b^2 - 2ab \cos C$ (***Law of Cosines***).

The law of cosines can be used to solve a triangle if you are given the measures of two sides and the included angle or three sides.

Example

Find x in the triangle shown below. Round your answer to the nearest 3 decimal places.



Example Solution

Fill in the law of cosines with the given information. Which side you choose for a and b doesn't matter but c should be the side opposite the given angle.

$x^2 = 17^2 + 33^2 - 2(27)(33) \cos 42^o$

When working with the law of cosines, it is important to remember the order of operations.

$x^2 = 289 + 1089 - 1324.284079$

$x^2 = 53.715921$

Square root both sides

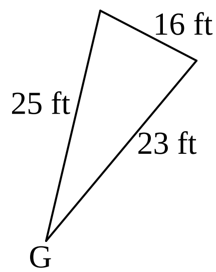
x = ± 7.323 (remembering to round to the nearest 3 decimal places)

Since we're talking about the measurement of the side of a triangle, our answer is

x = 7.323 yd

Example

Find the measure of angle G for the triangle shown below. Round your answer to the nearest 3 decimal places.



Example Solution

Fill in the law of cosines.

Which side you choose for a and b doesn't matter but c should be the side opposite the given angle.

$16^2 = 25^2 + 23^2 - 2(25)(23) \cos G$

$256 = 625 + 529 - 1150 \cos G$

Subtract 625 and 529 from both sides

$-898 = -1150 \cos G$

Divide both sides by -1150

$0.7808695652 = \cos G$

We're looking for an angle so we're going to use the inverse

$G = \cos^{-1}(0.7808695652)$

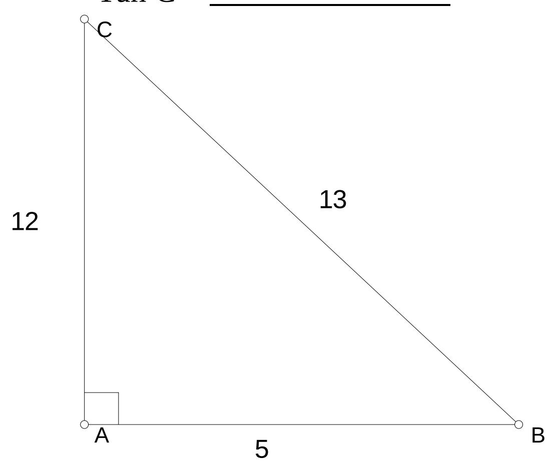
G = 38.660 (Remembering to round to the nearest 3 decimal places)

Practice Problems

1. Sin C = ________________

   Cos C = ________________

   Tan C = ________________



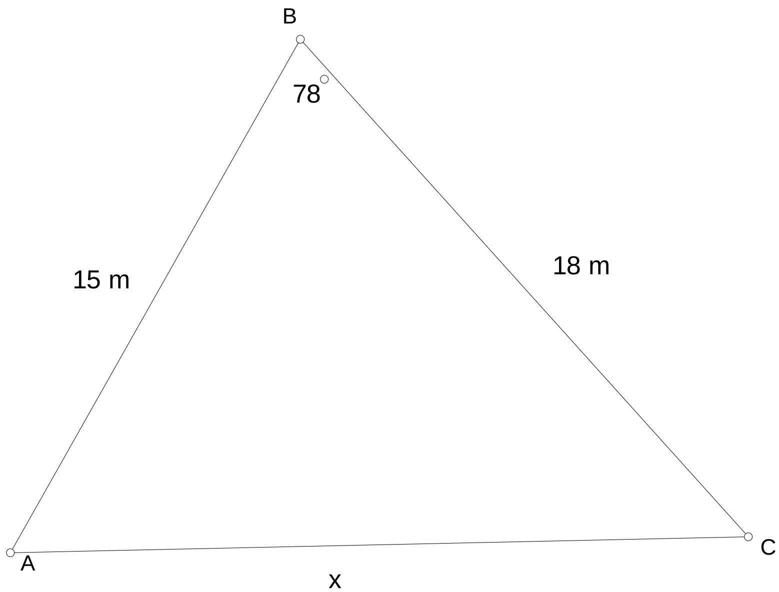
Use a scientific calculator to find the values to the nearest one decimal places.

2. Sin 2° =

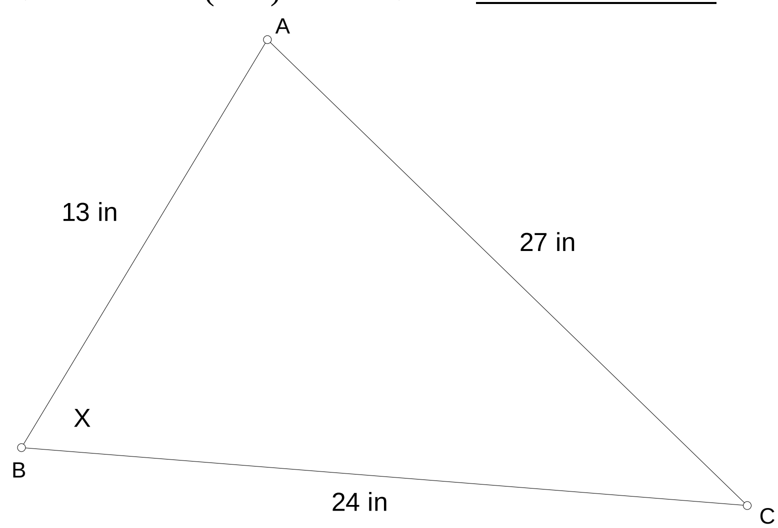
Find the measure of each angle to the nearest one decimal place.

3. cos B = .2376

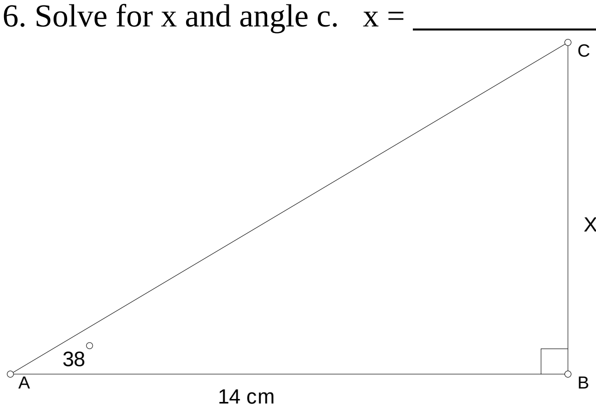
4. Solve for x. x = __________________ Find the area of triangle ABC =________________



5. Solve for x ($\angle B$) and $\angle C$. X = _______________ $\angle C$ = _______________



6. Solve for x and angle c. x = _____________ $\angle C$ = __________________



7. To the nearest degree, find the measure of angle B in ΔABC given the $m\angle A = 12°$, AC = 54 in, and BC = 40 in.

8. To the nearest meter, find the length of side DE in ΔDEF given $m\angle F = 55°$, DF = 15 cm, and EF = 8 cm.

9. The angle of elevation from a ship to the top of a 120 m lighthouse on the shore is $43°$. To the nearest tenth of a meter how far is the ship from the base of the lighthouse?

10. Sam is flying a kite with 175 ft. of string out. His kite makes an angle of $78°$ with the level ground. How high is the kite to the nearest foot?

11. How do you solve for θ, when $\sin\theta = \frac{\sqrt{2}}{2}$? What does θ equal?

Additional Concepts

Parallel lines
Perpendicular lines
Writing equations of lines
Triangle Properties
- Angles
- Sides
- Altitude
- Median

Distance formula