Holy Family Catholic High School

Algebra Review

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This packet will review some of the topics needed on the Holy Family Catholic High School mathematics placement tests. It is by no means complete nor should it be considered a substitute for taking an actual high school level Algebra I course. It is intended only for those that completed or are at least close to completing an Algebra I course. At times it may seem like it is insultingly easy but stick with it and I believe you will learn something.

One thing I also want to be clear about is that solving a problem is not the same as just getting the correct answer. That is like saying the only important thing about a roller coaster ride is where it ends. It really is how you get there that matters most. Okay I won't pretend that anyone reading this will think that solving a math problem is as much fun as a roller coaster though, perhaps, to some of you it is just as terrifying – in that case, perhaps I can help. Anyway, please pay attention to how the problem is solved and how the work is shown. When it finally does come to the test, you will be graded not just on the correct results but also how you solved your problems. That does not mean that it is do it my way or you automatically lose points. There are frequently multiple methods of solving a problem – just make sure yours is a valid **algebraic** method and not just something else like guess-n-check.

Unit 1: Conversions and Things

Time to get started but before we get to real algebra we will start with some basics which, by the way, you need to be able to do without a calculator.

Rounding

Frequently (especially in the real world – aka not school) answers to problems do not work out evenly. A lot of the time we really need the exact answer so answers are stuck being messed up disasters like $\sqrt{3}+\pi\sqrt{5}$. Have you ever tried cutting a piece of paper to be $\sqrt{3}+\pi\sqrt{5}$ inches long? Come on - Good luck! When the problem and situation calls for it – use rounding. Of course you are familiar with the concept but some people still make mistakes with it. For example, you could probably handle rounding to say 3 decimal places as in the following

3.4359 → 3.436   412.915389 → 412.915   .08403 → .084

You know the basic rules. For example, if the digit after what you want is a 5 or higher then round up, otherwise round down.

The question now is, were you aware that 5.4302 rounded to 3 decimal places is not 5.43. It is actually 5.430. I know you are probably thinking, *what is the difference?* If they were exact numbers than there wouldn't be any but as rounded off answers there is one. 5.43 would be from rounding off to two decimal places not three – in other words less accurate. Basically if the problem asks for a certain number of decimal places (unless it works out exactly to fewer digits) keep the zeroes there.

Now some problems for you to do *(don't blame me – I made this to be an optional packet)*
Round each of the following to the given number of decimal places

P 1-1 98.142 to 1 decimal place
P 1-2 9284.398 to 2 decimal places (careful – watch those zeroes)
P 1-3 342.5123 to 0 decimal places (or nearest integer – same thing)
P 1-4 .03464 to 4 decimal places.

Now it is time for number conversions. We are frequently given numbers in an inconvenient form, or the natural process of solving problems generates less-than-desirable (even ugly) results. These next sections are about being able to convert from one form of a number to another.

Fractions → Decimals

This is probably the easiest conversion. A fraction is simply another way of stating a division. $\frac{6}{2}$ is the same as saying $6 \div 2$. Both forms are equal to 3. So in order to convert a fraction, all you have to do is divide the top number by the bottom number. The thing to be aware of is that the conversion may not be always work out evenly. Sometimes when you try, it just won't end so you need to round. Look to see if the problem specified a certain number places. For example trying $\frac{7}{3}$ on a calculator will yield 2.33333333 the exact number of threes varies by calculator since in reality is goes on forever though there is an obvious pattern. You will later learn that any fraction can either be written as a terminating decimal (written exactly with set number of decimal places) or repeating (going on forever but it is just repeating the same sequence of numbers over and over again) Note that you may have heard of numbers like π which go on forever but do not ever repeat. π and numbers like it cannot be written as a fraction; hence, they are referred to as irrational numbers (cannot be written as a ratio of other numbers).

Example 1a

Convert $\frac{435}{13}$ to a decimal rounded to 2 decimal places

Solution: $435 \div 13 \approx 33.46$

Convert the fraction to decimal (round to 3 decimal places)

P 1-5 $\frac{2}{5}$ **P 1-6** $2/3$ **P 1-7** $\frac{11}{2}$ **P 1-8**) $\frac{13}{459}$

Decimals → Fraction

We will focus (for now) only on the terminating decimals such as .5 . These are relatively straightforward. First, figure out what place the last non-zero digit is in. In the case of .5 it is the tenths place. Then, place the shown number over that number so in this case $.5 = \frac{5}{10}$ although with that done you do need to try to reduce the fraction. Note that since 5 evenly divides both 5 and 10 we can reduce as follows $\frac{5}{10} = \frac{5 \div 5}{10 \div 5} = \frac{1}{2}$

Here is another example. .762 has the two in the thousands place so

$$.762 = \frac{762}{1000} = \frac{762 \div 2}{1000 \div 2} = \frac{381}{500}$$

Convert the decimal to a fraction (remember that you always reduce fractional answers)

P 1-9 .34 **P 1-10** .2 **P 1-11** .00982 **P 1-12** .1094

Improper Fraction → Mixed Number

Improper fractions are those that have a larger number on top (the numerator) than on the bottom (the denominator). To convert a number like $\frac{7}{3}$ you follow this process. First, divide the 7 by 3. Second, write that integer part (or whole part) in front of the fraction. The remainder of that division (the leftover part - in this case, a 1 since 3x2 only equals 6 so 7 has an extra 1) goes in the new numerator. The denominator stays the same. So, the result in this case is $2\frac{1}{3}$

Here is another example: $\frac{42}{5}$

First note that when dividing 40 by 5 we get an 8 but since 5x8 is only 40 there are still 2 left over which means: $\frac{42}{5} = 8\frac{2}{5}$

Convert the improper fraction to a mixed number

P 1-13 $\frac{24}{7}$ **P 1-14** $\frac{9}{5}$ **P 1-15** $\frac{125}{3}$ **P 1-16** $\frac{103}{5}$

Mixed Numbers → Improper Fraction

Mixed numbers are generally only useful as final answers. While solving, it is often easier to deal with a number when it is an improper fraction. For example, it is impossible to deal with the reciprocal of a number (useful when dividing) without first thinking of it as fraction (improper or otherwise). Say that you have $3\frac{4}{5}$ and want the improper fraction version. Multiply the integer part (the 3) by the denominator (5) and then add the numerator (4). Place the result over the denominator. In other words

$$3\frac{4}{5} = \frac{3 \cdot 5 + 4}{5} = \frac{19}{5}$$

Another example would be

$$2\frac{1}{3} = \frac{2 \cdot 3 + 1}{3} = \frac{7}{3}$$

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convert the mixed number to an improper fraction

P 1-17 $3\frac{2}{7}$ **P 1-18** $1\frac{11}{14}$ **P 1-19** $8\frac{3}{6}$ **P 1-20** $3\frac{7}{10}$

Decimal → Percent

This conversion merely requires moving the decimal point 2 places to the right. An example would be .567 = 56.7% The mistake most commonly made is to say that .2 = 2% when in fact .2 = 20%

Convert the decimal to percent

P 1-21 .3456 **P 1-22** .0518 **P 1-23** 2.35 **P 1-24** 100.54

Percent → Decimal

This is the opposite of the last topic. For example, changing the percentage 47.89% into decimal .4789 . The key here is to still make sure that you moved the decimal point two places – but now move it to the left. The only difference is you need to move it left. You might, for example, say that 4.2% = .42 when it should be 4.2% = .042%. Another version of this mistake is to say that 124% = .124 when in fact 124% =1.24

Convert the percent to decimal

P 1-25 24.5% **P 1-26** 26.9% **P 1-27** 142% **P 1-28** .3%

Exponents

All exponents are is a shortcut for writing out what could be long strings of multiplications. For example 3^4 could have just been written $3 \cdot 3 \cdot 3 \cdot 3$ The first format is just a lot easier to write (especially if I said 3^{100} which would otherwise require writing out a hundred 3's and quite frankly I don't have that kind of time). Before I start mentioning some of the quirks and basic facts, I should start with some terminology. Let's use 3^4 as a reference. In the expression, the 3 is known as the base. It is that thing which has the exponent or that number which is being multiplied repeatedly. Whatever you want to think of it as, just know that the big number is called the base. The other one which is written as a superscript (a fancy word but which breaks down easily. super = above and script = written so superscript just means written above) is just called the *exponent* (like that wasn't obvious) The exponent, in this case 4, reflects the number of times the number of times the number will be multiplied.

It might be useful to review what the true definition of an exponent is. x^y means to multiply 1 by x, y times. In reality then, $3^4 = 1 \cdot 3 \cdot 3 \cdot 3 \cdot 3$. The difference seems minor since putting the one in front of the 3's makes no difference in the value, but suppose we were to try 5^0. Saying this is just no 5's makes very little sense – because what number is it then. Now we can think of this as $5^0 = 1$. It is a 1 multiplied by 5 zero times. By putting the one in front of every exponent expansion we do not need to consider the 0 exponent as a different rule. Confused yet? You probably are but bear with me for a little while longer. Look at the following table.

2^0	1	1
2^1	$1\cdot 2$	2
2^2	$2\cdot 2$	4
2^3	$4\cdot 2$	8
2^4	$8\cdot 2$	16

Each time you increase the exponent by one you are simply multiplying the previous result by the base. Since it is only logical that $2^1 = 2$ the question becomes: *what* times 2 equals the desired number 2. Well 1 is the only valid answer. If we tried saying that $2^0 = 0$ we would have to conclude that $2^1 = 0\cdot 2 = 2$ which is really messed up.

Now that we have established the rule that each time the exponent is increased by 1 the previous answer is multiplied by the base we can consider the other direction. Our pattern *(notice I said "Our pattern" – I think it is supposed to make you feel like we are working together. Doesn't work does it?)* Anyway, our pattern (the one where $2^3 = 2^2 \cdot 2$) would suggest that $2^0 = 2^{-1} \cdot 2$. Since the answer is supposed to be 1 we are forced to ask what times 2 equals 1. ($x\cdot 2=1$) the only solution is that $2^{-1} = ½$. This process yields the following table

2^{-3}	¼ / 2	⅛
2^{-2}	½ / 2	¼
2^{-1}	½	½
2^0	1	1
2^1	$1\cdot 2$	2
2^2	$2\cdot 2$	4
2^3	$4\cdot 2$	8
2^4	$8\cdot 2$	16

You will hopefully have noticed the following $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ It seems that negative exponents evaluate just like their positive counterparts, except the results are now placed in the denominator under a numerator of 1.

By the way, realize that it usually does not matter what the base is when the exponent is 0. The result is nearly always 1. The only exception is 0^0. Logically, 0 to any power should be 0 but by our definition it is also true that anything to the 0 should be 1. 0^0 can not equal two things at once so we are forced to conclude that 0^0 is undefined. (That is the mathematician's way of saying – *I can't make any sense of this either so I am giving up.*)

This last section included some technical discussion and was probably confusing so I suggest reading it a few more times. Perhaps read it twice before trying the homework and once more after or, perhaps, in about a week.

Note that throughout this review I may give directions stating to use a certain number of non-zero decimal places. If I merely said 4 decimal places I would find students rounding (correctly

I might add) answers like .0001234567 to just .0001. While correct, this is not what I want. By saying "4 non-zero" decimal places my intention is to have the first three zeroes ignored and the counting to begin with the 1. The answer is therefore .0001234. Note that if a zero appears between numbers it should be counted. Only ones that do not follow a different number are skipped. So here are some answers given the way we want

$$.00892343 \approx .008923$$
$$2.03549 \approx 2.0355$$
$$.03549 \approx .03549$$
$$.210542 \approx .2105$$
$$.51 = .51$$
$$.412003 \approx .4120$$

Please get in the habit of using the approximately symbol whenever you are rounding off an answer. Also did you notice that I included the trailing zero in the last problem but did not add any in the one before it. Think back to the bit on rounding. It is necessary to have it there. Writing the answer as just .412 in a section requiring 4 places would have indicated that the answer was in fact *exactly* .412 when the answer in truth had to be rounded off.

Evaluate the given exponents – do this without a calculator since you may be required to do so on the test and or quiz

P 1-29 3^3 **P 1-30** 2^6 **P 1-31** 5^{-2} **P 1-32** 1^8

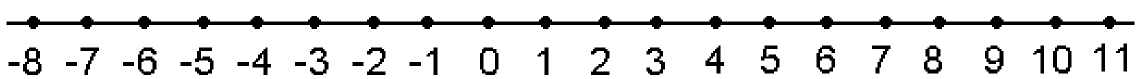
P 1-33 2^{-5} **P 1-34** 7^3 **P 1-35** 9^3 **P 1-36** 10^8

Unit 2: Absolutely Crazy

I am not going to spend much time on absolute value. Absolute value is just the function that turns whatever is inside into a non-negative number. To zero and positive numbers, the absolute value bars act just like parenthesis. Example: $|5| = (5)= 5$ and $|0| = (0) = 0$. Just remember to not apply the absolute value function to components inside the absolute value. It is only the result of everything inside that is made non-negative. To show you what I mean - here is a common error: $| 3 + -4 | = | 3 + 4 | = 7$. This is not – repeat - not correct. The correct simplification is $| 3 + -4 | = | -1| = 1$. Another error is to make the final result of the entire problem positive. For example: $-2 |3 - 5| = -2|-2| = -2 \cdot 2 = -4 = 4$ which is again WRONG. The correct process yields $-2 | 3 - 5| = -2 | -2| = -2 \cdot 2 = -4$. The final answer can be negative. It is only the inside of the absolute value that is forced to be a positive result (or zero – let's not forget zero).

Addition and subtraction

The first part of this review is about addition and subtraction. Those ideas are so basic that you may have forgotten how you learned them. Think back to first and second grade. You probably had a number line posted on the wall. That was how you learned addition and subtractions so we might as well go back to that. Let's use the following.



Pretend that each number is equally spaced out on the line (its pretty close anyway). Say that you were given the problem of $4 + 3$ That means that you start out standing at 4 and then take three steps to the right. You would find yourself at 7 so $4 + 3 = 7$. I know this is simple but just stay with me -- unless you have a headache in which case you should probably skip the next 3 paragraphs.

In general, we always think of the addition of two numbers $(a + b)$ as starting out at the first number (a) and then taking b steps to the right. This works even if we throw in problems such as $-5 + 8$. We start at -5 and take 8 steps to the right to end up at 3. (in other words $-5 + 8 = 3$) Now we need to deal with things like $8 + -5$. It is possible to always think of a negative as meaning the opposite direction. We are either starting in the opposite direction from 0 or we are moving in the opposite direction from normal addition which is to the right. This means that $8 + -5$ means to start at 8 and then instead of moving to the right 5 we move to the left 5. When you move 5 to the left you end up at 3 so $8 + -5 = 3$.

Of course, you may have noticed that I have only talked about addition. That is because, in one way of looking at things, subtraction is not a different operation. Technically subtraction is defined as $a - b = a + -b$ so subtraction is just a way to write an addition problem.

IMPORTANT: Note that you should also learn not to think of a "-" as a negative sign. It is either a subtraction symbol, when placed between two symbols or an opposite sign when placed in front of a number or variable. While this appears to be a subtle and perhaps insignificant difference, it is important. People frequently make a mistake because they think of "-b" as negative b when it really means the opposite of b. They assume that "-b" is a negative number when that is only true when b is positive to start with. If b is a negative number than "-b" is the opposite- a positive number.

Back to subtraction, 9 – 4 means 9 + -4. Simple enough but you could have handled that with your old way of thinking about subtraction. Look at a problem like 9 - -4 You probably learned an arbitrary rule to handle this but since we now see subtraction as adding the opposite we just need to know that the opposite of –4 is a positive 4 to state the following:
9 - -4 = 9 + (the opposite of –4) = 9 + 4 = 13.

REALLY REALLY REALLY IMPORTANT: Yes, it is true that $x - 5 = x + -5$ and there are times when the + - 5 concept is very helpful while solving a problem; still, it is never acceptable as part of your final answer. If you are getting x + - 3 than your answer **must** be x – 3.

Terminology sidebar:

I referred to the opposite of a number earlier. This is somewhat sloppy terminology. It worked since I was dealing with the context of addition. When you want opposite to mean things like 2 and –2 then the correct term is "additive inverse." Think for a moment about what a number and its additive inverse add up to.

An identity is a number, which, for that operation, returns the starting number. Using the a + b notation, what would b have to be in order for the sum to always be a. For example, 7 + [] = 7. What needs to go in place of [] to make it work out? Hopefully, you know it is a 0. In fact a + 0 = a no matter what a is. This means that 0 is the additive identity. As it turns out, since 0 is its own additive inverse, it is also the subtraction identity, but that term is almost never used. When, in the last paragraph, I asked you to think about the addition result of a number and its additive inverse you, hopefully, realized that the result is always the additive identity, zero. Coincidence? Not really.

P 2-1 What is the additive identity

P 2-2 What is the additive inverse of 5

P 2-3 What is the additive inverse of ½

P 2-4 How is subtraction defined?

P 2-5 | 2 – 10|

P 2-6 3 + |4 + 5|

P 2-7 5 - | 10 – 2 |

P 2-8 2 + 14

P 2-9 9 + -4

P 2-10 26 – 5

Multiplication

I probably do not have to explain what multiplication is but as long as I am explaining everything else I might as well. In its most basic form multiplication means add the first number as many times as the second number.

Example $5 \cdot 6 = 5 \textit{ added six times} = 5+5+5+5+5+5 = 30$

Of course this is only the base definition since it does a poor job explaining how to handle problems like $\frac{2}{5} \cdot \frac{7}{3}$ Remembering that 7/3 = 2 1/3, It still makes very little sense to say this means $\frac{2}{5}+\frac{2}{5}+\ldots$ but we will deal with fractions later.

I just want to remind you of a few basic facts

1) Communitive property	a · b = b · a	3 · 4 = 4 · 3
2) Associative property	(a · b) · c = a · (b · c)	(2 · 3) · 4 = 2 · (3 · 4)
3) Multiplication by zero	a · 0 = 0 for any value of a	12345 · 0 = 0
4) Identity of multiplication = 1	a · 1 = a for any value of a	5 · 1 = 5

You, hopefully, remember that we talked about identities earlier during addition. It was whatever, for the given operation, yields the original number. We also noted that adding the additive inverse yields the additive identity. That suggests that the multiplicative inverses should multiply to equal the multiplicative inverse, 1. What, then, is the multiplicative inverse of numbers like 3? What would need to replace [] in the equation 3 · [] = 1 to make it a valid equation? The only solution is what we call the reciprocal, $\frac{1}{3}$. For any number, the multiplicative inverse or reciprocal is the fraction written with the numerator and denominator in the opposite positions. Numbers not written as fractions should just be thought of as over 1. So-

$$\frac{6}{9} \to \frac{9}{6} \text{ since } \frac{6}{9} \cdot \frac{9}{6} = 1$$

$$7 = \frac{7}{1} \to \frac{1}{7} \text{ since } \frac{7}{1} \cdot \frac{1}{7} = 1$$

Also remember the following general rules

positive · positive = positive + + → +
positive · negative = negative + - → -
negative · positive = negative - + → -
negative · negative = positive - - → +

P 2-11 What is the multiplicative inverse of 5

P 2-12 What is the multiplicative inverse of ½

Simplify

P 2-13 41 – 12 + 6 – 10

P 2-14 5 – 1 –4 –6 + 1

P 2-15 5 · 4

P 2-16 6 · 14

P 2-17 4 · 6 · 1 – 5 + 2

Matrices

Matrices are a structured way to express a collection of data (or equations as we will eventually learn in some other math class) in a simple box form. For example, I could list a price table as small t-shirt are $12.5 and medium t-shirts are $14.00 and large t-Shirts are $15.36 and small polo shirts are $26.21 and medium polo shirts are $26.75 and large polo shirts are $26.99. This same data could be written as

$$\begin{array}{cc} & \begin{array}{ccc} small & medium & l\arg e \end{array} \\ \begin{array}{c} T-shirt \\ polo \end{array} & \begin{bmatrix} \$12.50 & \$14.00 & \$15.36 \\ \$26.21 & \$26.75 & \$26.99 \end{bmatrix} \end{array}$$

You would probably agree that the matrix form is a lot easier to deal with than the long list of prices. Imagine adding more categories of items and more sizes and the difference becomes even more obvious.

There are times when matrices are simply data storage as in the previous example but, depending on what information is stored there, it sometimes make sense to perform operations with them. For example

$$\begin{bmatrix} 4 & 2 \\ 1 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 5 \\ 2 & 10 \end{bmatrix}$$

The way to handle both addition and subtraction is to perform the operation on numbers that are in the same position – first row first column to first row first column. So the previous is

$$\begin{bmatrix} 4 & 2 \\ 1 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 5 \\ 2 & 10 \end{bmatrix} = \begin{bmatrix} 4+-3 & 2+5 \\ 1+2 & 9+10 \end{bmatrix} = \begin{bmatrix} 1 & 7 \\ 3 & 19 \end{bmatrix}$$

Similarly

$$\begin{bmatrix}1 & 5\\ 6 & 9\end{bmatrix} - \begin{bmatrix}3 & -1\\ 2 & 4\end{bmatrix} = \begin{bmatrix}1-3 & 5--1\\ 6-2 & 9-4\end{bmatrix} = \begin{bmatrix}-2 & 6\\ 4 & 5\end{bmatrix}$$

Or

$$\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix} + \begin{bmatrix}2 & 9 & 7\\ 5 & -2 & -1\end{bmatrix} = \begin{bmatrix}1+2 & 2+9 & 3+7\\ 4+5 & 5+-2 & 6+-1\end{bmatrix} = \begin{bmatrix}3 & 11 & 10\\ 9 & 3 & 5\end{bmatrix}$$

However, what if I wanted you to do the following

$$\begin{bmatrix}5 & 4 & 8\\ 2 & 8 & 6\end{bmatrix} - \begin{bmatrix}3 & -1\\ 2 & 4\end{bmatrix}$$

Those two matrices don't match which actually makes this a really easy problem – all you do is say "not possible". You can only add or subtract matrices when they are the same size.

Simplify the following:

P 2-18 $\begin{bmatrix}3 & -2\\ 5 & 8\end{bmatrix} + \begin{bmatrix}10 & 7\\ 2 & 3\end{bmatrix}$

P 2-19 $\begin{bmatrix}1 & 6\\ 5 & 9\\ 4 & 2\end{bmatrix} + \begin{bmatrix}5 & 2 & -9\\ 7 & 6 & 4\end{bmatrix}$

P 2-20 $\begin{bmatrix}1 & -2 & 3\\ -4 & 5 & -6\\ 7 & -8 & 9\end{bmatrix} - \begin{bmatrix}10 & 7 & 5\\ 4 & -2 & -1\\ 11 & -8 & -10\end{bmatrix}$

P 2-21 $\begin{bmatrix}4 & 2\\ -1 & 25\end{bmatrix} - \begin{bmatrix}5 & 6\\ 7 & 8\end{bmatrix} + \begin{bmatrix}-2 & -3\\ 8 & 9\end{bmatrix}$

Distributive property

Originally, problems like 4(x+5) could not have anything done without knowing the actual value of x. The distributive property lets us do something with it even though by itself we still cannot do much. The distributive property says that a(b + c) = ab + ac. Keeping in mind the definition of multiplication (ab = a +a + a .. + a b times or ab = b + b+b+...+b a times)

$$a(b+c)$$
$$(b+c)+(b+c)+..(b+c) \quad a\ times$$
$$b+c+b+c+...b+c$$
$$b+b+..b+c+c+..c \quad each\ a\ times$$
$$ab+ac$$

Or more concrete

$$4(x+5)$$
$$(x+5)+(x+5)+(x+5)+(x+5)$$
$$x+5+x+5+x+5+x+5$$
$$x+x+x+x+5+5+5+5$$
$$4x+4\cdot 5$$
$$4x+20$$

This means that if there is a multiplication by a quantity inside parenthesis then it is possible just to multiply each part inside individually. By each part I mean each term separated by an addition or subtraction. It is not correct to say that $2(4x+5)=2\cdot 4\cdot 2x+2\cdot 5=16x+10$ because the 4 and x are separated by multiplication. The correct simplification is $2(4x+5)=8x+10$. Part of the power of the distributive property is actually in using it in the other direction. For example since it is true to say that $3(5x+2)=15x+6$ we must also be able to say that $15x+6=3(5x+2)$. So when faced with something like $40x+24$ we can say that it is equal to $8(5x+3)$ by using the distributive property although we often call this factoring. This property can often help us solve problems. It is not even limited simply to having numbers outside the parenthesis. It is also true that $5x(2x+8) = 5x\cdot 2x + 5x\cdot 8 = 10x^2 + 40x$

P 2-22 $5\,|\,40 - 28\,|$

P 2-23 $5\,|\,2 - 10\,|$

P 2-24 $4(x+10)$

P 2-25 $5x(2x-14)+20x$

P 2-26 $5 - 2(6x+13)$

P 2-27 $10+2 - 5 + 6(4 - 1)$

P 2-28 $15 + 2x + (3x - 20)\ 4$

Unit 3: Fractured Fun

Division

Previously we dealt with addition and subtraction and finally multiplication. Division did not come up as a real issue (not counting that fraction conversion). In truth, the operation did come up in a different way. You may remember my saying that subtraction is merely addition (subtraction = addition of the additive inverse). In the same way, division may be defined as just a multiplication and, as before, by the inverse only this time it is the multiplicative inverse. The multiplicative inverse or reciprocal of a numbers, as it is usually called, is the number which when multiplied by the original number yields a product equal to one. For example 1/3 is the reciprocal of 3 since $3 \cdot 1/3 = 1$. Of course as this example shows, it is often easier to figure out the reciprocal than the technical definition might suggest. All you have to do is place it as the denominator of a fraction with 1 as the numerator. Of course this would make no sense if the number started out as a fraction so here you just flip the number over – numerator to denominator and vice-versa.

Part of the importance of defining division this way (as opposed to a definition involving something like how many of b fit in a for a/b) is that now we can basically say that anything that works for multiplication also works for division (the primary exception being that 0 is a mess for division. It also is a great help when dividing by fractions. Consider the problem of $\frac{3}{4} \div \frac{5}{7}$.

Try figuring out how many five seventh's fit into three fourths and you will probably give yourself a headache. When division can be defined as multiplication by the reciprocal it is not that bad. You would solve it this way

the reciprocal of $\frac{5}{7}$ is $\frac{7}{5}$ so

$$\frac{3}{4} \div \frac{5}{7} = \frac{3}{4} \cdot \frac{7}{5} = \frac{21}{20}$$

A couple things to be wary of

Say that you are given a problem like $\frac{6x+5}{3}$, you may be tempted to say 2x + 5. *(hey look x's we have to be close to algebra)* You hopefully are used to being able to reduce $\frac{6x}{3}$ to 2x but it does not work to do so in the original problem. Perhaps the best way to explain why this does not work is to again use the definition of division. We will use the fact that the reciprocal of 3 is 1/3.

$$\frac{6x+5}{3} = (6x+5)\frac{1}{3} = \frac{1}{3}(6x+5) \qquad \text{by definition of division}$$

$$= \frac{1}{3} \cdot 6x + \frac{1}{3} \cdot 5 \qquad \text{by the distributive property}$$

$$= \frac{1}{3} \cdot \frac{6}{1} x + \frac{1}{3} \cdot \frac{5}{1} \qquad \text{any number may be placed of 1}$$

$$= 2x + \frac{5}{3} \qquad \text{multiplication of fraction}$$

The other thing is just how do deal with fractions so that is our next topic

Fraction operations

You have already been taught how to handle fractions, but this is only a review. If this quick lesson does not make sense than make sure you seek help soon. Learning to handle fractions can a big hurdle to get over but once you do, you will realize that they are not so bad

Addition

The key to fraction addition is common denominators. That is to say that the only way to add fractions together is if they have the same denominator. If you were putting together rolls of coins, you would want every coin in a roll to be the same type. If you try to put together a roll of quarters with dimes mixed it, it would not work very well. It is about the same to try to do $\frac{2}{3} + \frac{1}{2}$ without a common denominator. Keep in mind that the denominator basically tells you what size pieces you have and the numerator is how many of those pieces you have. For example, the $\frac{2}{3}$ can be thought of having taken a cake(or pizza if you prefer) which was divided into three parts of which you have 2. Since the other cake was divided into five pieces it is hard to say exactly how much you have since the pieces are different sizes. They do not fit together. But what if they were all one-sixth of a cake. Imagine cutting each of the thirds in half. You would now have 4 pieces all of which are the same size as if the cake had been cut into 6 pieces originally.

$$\text{Mathematically } \frac{2}{3} = \frac{2}{3} \cdot \frac{2}{2} = \frac{2 \cdot 2}{3 \cdot 2} = \frac{4}{6}$$

The 1 piece which was half a cake can be cut into three pieces which would yield pieces that are one-sixth the size of the original cake.

$$\frac{1}{2} = \frac{1}{2} \cdot \frac{3}{3} = \frac{1 \cdot 3}{2 \cdot 3} = \frac{3}{6}$$

So finally we can put is all together since all pieces are the same sizes.

$$\frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6} = \frac{7}{6}$$

Here is another example without all the narration

$$\frac{7}{12} + \frac{3}{5} = \frac{7}{12} \cdot \frac{5}{5} + \frac{3}{5} \cdot \frac{12}{12} = \frac{35}{60} + \frac{36}{60} = \frac{71}{60}$$

Hopefully you noticed a few things. First, that the basic process involved multiplying both the top and bottom by the same number. With top and bottom the same the fraction equals 1 and as you already learned, multiplication by one does not change anything. Second, in the final addition step, I only added the numerators and left the denominator alone. A common mistake people make is to write 71/120 but you do **not** add denominators.

Now look at the following $\frac{4}{6}+\frac{3}{8}$. It can be done several equivalent ways.

$$\frac{4}{6}+\frac{3}{8}=\frac{4}{6}\cdot\frac{4}{4}+\frac{3}{8}\cdot\frac{3}{3}=\frac{16}{24}+\frac{9}{24}=\frac{25}{24}$$

or

$$\frac{4}{6}+\frac{3}{8}=\frac{4}{6}\cdot\frac{8}{8}+\frac{3}{8}\cdot\frac{6}{6}=\frac{32}{48}+\frac{18}{48}=\frac{50}{48}=\frac{50\div 2}{48\div 2}=\frac{25}{24}$$

In the first example I used what is called the least common denominator (the smallest number which is divisible by both denominators.) while the second time I just multiplied each fractions top and bottom number by the denominator of the other fraction. This second way required the extra step of reducing the fraction so it seem like an odd way to do it. In truth it does not matter which way you do it. Sometimes the extra time it would take to figure out what is the least common denominator is more than it takes to reduce the fraction which sometimes has to be done even if you did find the least common denominator.

Subtraction

The same as addition except that you subtract the numerators after you get a common denominator.

***Please be aware that I expect you to be able to do addition and subtraction on your own so throughout this year I expect to always see the work (ie, showing the fractions with common denominator before you write the final result)

Multiplication

This is usually a lot easier than addition or subtraction. Addition and subtraction require the somewhat messy and frequently error prone process of common denominators. Multiplication does not require this. For multiplication you just need to multiply across numerator times numerator and denominator times denominator. So here is a typical case:

$$\frac{3}{8}\cdot\frac{7}{5}=\frac{3\cdot 7}{8\cdot 5}=\frac{21}{40}$$

While at times it is necessary to reduce the final outcome, it really never gets more difficult than that. Here is a number of examples just to justify having a section on multiplication.

$$\frac{5}{2}\cdot\frac{5}{3}=\frac{25}{6}$$

$$\frac{9}{2}\cdot\frac{4}{3}=\frac{36}{6}=6$$

$$\frac{12}{5}\cdot\frac{15}{7}=\frac{180}{35}=\frac{180\div 5}{35\div 5}=\frac{36}{7}$$

$$\frac{8}{12}\cdot 6=\frac{8}{12}\cdot\frac{6}{1}=\frac{48}{12}=4$$

$$\frac{5}{2}\cdot\left(3\frac{4}{5}\right)=\frac{5}{2}\cdot\frac{19}{5}=\frac{95}{10}=\frac{19}{2}$$

Note that you must always reduce your final answer!!! Also, never leave anything as a number over one in your final answer you write 6/1 as just 6. Please note that it would have been very easy to interpret the $3\frac{4}{5}$ as 3 times 4/5 which is why mixed numbers should never show up in problems except as a final answer.

Division

Division is relatively simple once you know the trick. I actually covered it earlier but I thought this would be a good time to point out a few things. Remember that division can be rewritten as just the multiplication by the reciprocal. Look at the following example of how this is done.

$$\frac{5}{4}\div\frac{3}{7}=\frac{5}{4}\cdot\frac{7}{3}=\frac{35}{12}$$

A few things are worth pointing out. First note that only the second of the pair (the one being divided by) is flipped over. The first number is left alone. It is not a case of just flip one over and it does not matter which. You always leave the first alone and take the reciprocal of the second. Of course you may already be aware that division shows up in different forms so take a look at this

$$\frac{21}{\frac{3}{4}}\ \textit{means}\ 21\div\frac{3}{4}=\frac{21}{1}\div\frac{3}{4}=\frac{21}{1}\cdot\frac{4}{3}=\frac{84}{3}=28$$

Again make sure you always reduce your final answer as much as possible.

P 3-1 $\frac{3}{4}+\frac{7}{3}$

P 3-2 $\frac{3}{4}-\frac{7}{3}$

P 3-3 $\frac{3}{4}\cdot\frac{7}{3}$

P 3-4 $\frac{3}{4}\div\frac{7}{3}$

P 3-5 $\frac{3}{8}+7$

P 3-6 $\frac{3}{8}-7$

P 3-7 $\frac{3}{8}\cdot 7$

P 3-8 $\frac{3}{8}\div 7$

P 3-9 $\frac{3}{4}+\frac{14}{13}-\frac{1}{2}$

P 3-10 $\frac{10}{9}\div\frac{3}{5}$

P 3-11 $\dfrac{\frac{1}{4}+\frac{2}{11}}{\frac{3}{4}}$

Unit 4: Single Transformation

We will first deal with simple cases in which only one operation is required to solve the equation. Baring the use of more advanced functions, there are only four types that we need to worry about: addition, subtraction, multiplication, and division. In a sense however there is not even that many different things to worry about. As we will see, each case boils down to finding a way to cancel out the unwanted part. We then need to make sure that we do the same thing to both sides. By doing this we are assured that even though we are changing the appearance of the equation we are not actually changing situation. For example, suppose we had the rather ridiculous equation of 4=4. It is obviously true so hopefully no arguments so far. Now if I were to add three to just one side we would get 4+3 = 4 or 7 = 4 which of course makes no sense whatsoever. If we expect to get an equation that makes sense we need to add the same things to both sides. 4+3 = 4 + 3 to get 7 =7. This idea of doing the same things to both sides is usually referred to as keeping the equation balanced.

Addition x+a=b

Our goal here is get the variable x by itself. In most cases, at least in the beginning, the a and b are actually just numbers. For example, x + 4 = 10 is a typical problem. One way to think about the solving of this problem is to ask what cancels a +4. Well since the 4 - 4 = 0, the answer is a –4 and the solving of the problem is done to match.

$$x+4=10$$

$$x+4-4=10-4$$

$$x=6$$

Here is another example

$$x+5=12$$

$$x+5-5=12-5$$

$$x=7$$

In reality it does not matter what the number being added to x. It does not have to even be a definite number. Even if it is just a variable the method of solution is still the same.

$$x+a=10$$

$$x+a-a=10-a$$

$$x=10-a$$

While we can not actually perform the operation of 10 – a, we have solved the equation for x which was our goal.

What if the equation looks a little different - such as 3 + x=9. We solve it same way although it may help to remember the basic rule, the Communitive property of addition which says a + b = b + a or 3+ 4 = 4 + 3. So here is the solution to our problem

$$3+x=9$$

$$x+3=9$$

$$x+3-3=9-3$$

$$x=6$$

Actually we rarely ever visibly switch the three and x around but until you are confident about what you are doing, feel free to actually do so.

Subtraction x-a=b

(some of the wording in this section will probably seem familiar)

Once again we want to get x by itself and again the a and b are usually just numbers. The only real difference is that now we are forced to ask what cancel out a minus number. Lets use x + 4 = 10 as our typical problem. Well since –4 and 4 are additive inverses (i.e. -4 + 4 = 0), the answer is to add 4.

$$x - 4 = 10$$
$$x - 4 + 4 = 10 + 4$$
$$x = 14$$

Here is another example

$$x - 5 = 12$$
$$x - 5 + 5 = 12 + 5$$
$$x = 17$$

Again it does not matter what the number being subtracted from x. It does not have to even be a definite number. Even if it is just a variable the method of solution is still the same.

$$x - a = 10$$
$$x - a + a = 10 + a$$
$$x = 10 + a$$

While we can not actually perform the operation of 10+ a, we have solved the equation for x which was our goal.

What if the equation looks a little different - such as 3 – x = 9. The last time we used the Communitive property to switch the x and 3 around but we can not do it here since there is no Communitive property of subtraction. Why not? Well look at this: 9 – 3 = 6 while 3 – 9 = -6. We get the opposite number if we try to switch them around. We will deal with solving this type of problem later since it is possible to rewrite the problem as a two step problem(3 + -1·x=9) and we only wanted to handle single step problems for now.

Hold on a second!

You probably did not realize it yet but I glossed over a type of problem in both of the last two sections. What if you have x + -5 = 8. There are actually two ways of handling this problem.

You could handle it like an addition problem. We basically learned there to subtract the number that was being added and that works.

$$x + -5 = 8$$
$$x + -5 - -5 = 9 - -5$$
$$x = 9 + 5$$
$$x = 14$$

That may seem kind of messy so most people prefer to remember the definition of subtraction which says that x – 5 = x + -5 except we will use it in reverse

$$x + -5 = 8$$
$$x - 5 = 8$$
$$x - 5 + 5 = 8 + 5$$
$$x = 13$$

Which way you deal with it is up to you. Just make sure you whatever you have on the left side does cancel out to give just x (or m or p or whatever variable is used.

Leave exact answers (use decimals only if they are exact and reduce all fractions)
Do these problems without a calculator and as always – show your work
Solve for the variable   -- if more than one solve for x

P 4-1 x + 9 = 15
P 4-2 x –12 = 41
P 4-3 9 + x = 15
P 4-4 r – 10 = 20
P 4-5 x + 1 = 14
P 4-6 x + -3 = 12
P 4-7 b+ ½ =5
P 4-8 $x - \frac{2}{3} = \frac{4}{9}$
P 4-9 $x - \frac{4}{5} = \frac{3}{5}$
P 4-10 x - -3 = 16
P 4-11 x + a = 6
P 4-12 x – b = d

Multiplication x·3=12

In some ways this is actually easier. What is the opposite of multiplying by 3 – dividing by three is the somewhat natural answer so here is the solution.

$$x \cdot 3 = 12$$
$$x \cdot 3 \div 3 = 12 \div 3$$
$$x = 4$$

Of course it is my nature to complicate things so I am also going to offer an alternative solution method.   Remember multiplicative inverses, otherwise known as reciprocals? They make some problems easier. First lets redo the last one.

$$x \cdot 3 = 12$$
$$x \cdot 3 \cdot \frac{1}{3} = 12 \cdot \frac{1}{3}$$
$$x = 4$$

So why bother with this. Look at the following problem

$$x \cdot \frac{4}{5} = 12$$
$$x \cdot \frac{4}{5} \div \frac{4}{5} = 12 \div \frac{4}{5}$$
$$x = 12 \cdot \frac{5}{4}$$
$$x = \frac{60}{4}$$
$$x = 15$$

This is basically a mess.
Try it this way now.

$$x \cdot \frac{4}{5} = 12$$
$$x \cdot \frac{4}{5} \cdot \frac{5}{4} = 12 \cdot \frac{5}{4}$$
$$x = \frac{60}{4}$$
$$x = 15$$

Not a big improvement but, once you are used to it, it does get better – at least in cases involving fractions.

By the way note that there is a Communitive property for multiplication so solving x·3=12 is the same as solving 3x=12. Either way you divide by 3.

Division x÷a=b or $\frac{x}{a} = b$

Well this is actually pretty simple since what is the opposite of dividing something into pieces – putting it together. Say what?
Well do not worry about that but just think for a moment. What is the opposite of dividing? The answer is of course multiplying.

$$x \div 3 = 12$$
$$x \div 3 \cdot 3 = 12 \cdot 3$$
$$x = 36$$

of course the same problem could have been written differently but the solution is the same

$$\frac{x}{3} = 12$$
$$\frac{x}{3} \cdot 3 = 12 \cdot 3$$
$$x = 36$$

Please note that 3÷x=12 is a very different problem and just like 5-x is not a one step problem

Summary

So to review single operations here is a quick chart

type	*example*	*idea*	*shown*	*answer*
$+$	$x+3=12$	$-$	$x+3-3=12-3$	$x=9$
$-$	$x-3=12$	$+$	$x-3+3=12+3$	$x=15$
$\cdot$	$x\cdot 3=12$	$\div$	$x\cdot 3\div 3=12\div 3$	$x=4$
$\div$	$x\div 3=12$	$\cdot$	$x\div 3\cdot 3=12\cdot 3$	$x=36$

I can get by just knowing to always do the opposite operation with the same number as you start with. You can also use the same operation with the opposite number (meaning the inverse – additive for addition or subtraction and multiplicative, reciprocal, for multiplication or division

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Same directions as last ones

P 4-13 m · 3 = 14

P 4-14 p÷3 = 15

P 4-15 5x = 12

P 4-16 $\frac{4}{7}x=\frac{5}{3}$

P 4-17 $x\div\frac{5}{8}=\frac{7}{4}$

P 4-18 $w\cdot 4=\frac{1}{3}$

P 4-19 x ÷t=k

P 4-20 ax=4

P 4-21 bx =p

P 4-22 Demonstrate both ways of solving this problem -- using division and multiplying by reciprocal

$$\frac{2}{3}x=5$$

Unit 5: Solving Equation complications

Last time we just dealt with very simple equations. All you had to do was one simple operation in order to solve it. This time we need to look at slightly more complicated problems. For now we will still avoid more complex matters such as higher powers and square roots but we do need to deal with cases that involve two or more operations. It may be an operation on one side or it may be that two things need to be both sides rather than just one.

Combing Like Terms

At times it may be that you need combine terms on one side before dealing with crossing values from one side to the other. You probably, or at least hopefully, referred to terms which could be combined together as like terms. A term is just a less fancy name for what is technically called a monomial. A monomial consists of elements joined only by multiplications. It could be just a number, a variable, or maybe both. It may even be the case that there are multiple variables in a monomial. It may be repetitive but it is important to say again that it can not have any operation other than multiplication. Here are some examples to clarify.

these are monomials – 3, x , 5x, x^3, $7x^2$, $9xy^2$, $14xy^2z^3w^4$, $\frac{3}{2}x^4$

these are not monomial - 3 + 5x, 5x/2 , 9 – x, 3x + 2y

Monomial terms are said to be like terms if they have the exact same variables and each variable has the same exponent. This means that 4x and 8x are like terms since both have only x and each x has a power or 1. We can therefore take the expression 4x + 7x and say it equals 11x. On the other hand, if we were given $4x + 7x^2$ or 3x + 7y then there is nothing you can do with them. In the first case, the terms each have x as the only variable but the exponents do not match. The second case has exponents of 1 for both terms but one has x and the other y.

Even when people understand the idea of matching up variables they often make one mistake. There is a tendency to say that 5 + 4x equals 9x even though 5 and 4x do not have the same variables and thus are not like terms. In fact it is easy to prove that this did not work. To test if an operation does or does not work we can simply try a value for x. In this case, lets us x equals 2. So 5 + 4x = 5 + 4(2) = 5 + 8 = 13 while 9x = 9(2) = 18. While it is possible for one particular value to appear to work, any time we find any value of x which shows the two are not equivalent than we know it is not correct. Be warned that if the two had come out to the same thing we could only say we have evidence which suggests that it might be true while even one counterexample proves something is false.

So it is about time we get some actual problems

$$3x + 7x = 16$$

$$10x = 16$$

$$\frac{1}{10}10x = \frac{1}{10}16$$

$$x = \frac{8}{5}$$

It is hardly any different than what we had in the last unit. There is just the extra step of adding 3x to 7x.

Two or more transformations

The real focus of this packet is on solving equations for with two or more transformations are required to solve it. To make sure you know what I am talking about, look at the following

$$3x + 6 = 18$$

If it were just 3x = 18 you would divide by three. If it were just x + 6 = 18 you would subtract 6. Since both are there, you need to actually do both operations in order to solve the problem. The only question is which do you do first? Imagine for a moment that you had 3x +6 and had just learned x = 5. You would then have $3 \cdot 5+6$. According to the order of operations, to simplify this you would multiply the 5 by the 3 and then add 6. In some ways you can think of solving equations as going through the order of operations in reverse order. You deal with the 6 and then with the 3.

Example 1:

$$3x + 6 = 18$$
$$3x + 6 - 6 = 18 - 6 \qquad \textit{first subtract } 6 \textit{ from both sides}$$
$$3x = 12$$
$$\frac{3x}{3} = \frac{12}{3} \qquad \textit{divide both by } 3$$
$$x = 4$$

Another way of thinking about it is to always deal with the part least closely connected to the variable we are solving for first.

Example 2:

$$9x - 4 = 13$$
$$9x - 4 + 4 = 13 + 4 \qquad \textit{first subtract } 6 \textit{ from both sides}$$
$$9x = 17$$
$$\frac{9x}{9} = \frac{17}{9} \qquad \textit{divide both by } 3$$
$$x = \frac{17}{9}$$

Example 3:

$$\frac{x+4}{5} = 10$$
$$5 \cdot \frac{x+4}{5} = 5 \cdot 10$$
$$x + 4 = 50$$
$$x + 4 - 4 = 50 - 4$$
$$x = 46$$

Example 4:

$$7(3x + 4) = 21$$
$$3x + 4 = 3 \quad \textit{divide by } 7$$
$$3x = -1 \quad \textit{subtract } 4$$
$$x = \frac{-1}{3} \quad \textit{divide by } 3$$

This last example also demonstrates the minimum amount of work you should be showing. What your rules were during Algebra is hard to say but at this point you can start showing a little less. You do not need to actually write the –4's when subtracting 4 from each side. You just show a new line where the four has been subtracted. You should however note that you still do one step at a time. No subtracting by 4 and dividing by three in the same step.

Example 4 also shows something for you to think about. You may have noticed that you could have distributed the seven through the parenthesis and then dealt with 21x+28=21. This works about the same. It does not matter which you do. In fact, if the seven did not divide evenly into the 21 than it would have been better to distribute through.

Which Side?

There will be times when you are not sure which way to do things because you will have x on both sides. The answer of which side to work with is really easy – it does not matter. Although we prefer that final answers place x on the left (such as x = 5), it does not matter which way you solve it. The idea is just to make sure **all** x's (or whatever other variable you are solving for) are on one side while all other numbers and variables are on the other. For example, suppose you are given

$$4x + 2 = 9x - 6$$

You actually have quite a few options on problems like this. Lets suppose that you decide to move x's to the left first. You subtract 9x from each side to get

$$-5x+2=-6$$

At this point it is just like the problems from the last section, a two transformation solving. Here is the full solution

Example 5:

$$4x + 2 = 9x - 6$$
$$-5x + 2 = -6$$
$$-5x = -8$$
$$x = \frac{-8}{-5}$$
$$x = \frac{8}{5}$$

Of course you would end up with the same thing if you had subtracted 4x from each side and then solved the problem 2 = 5x – 6.

Some people always prefer to solve by putting the variable on the left side while others say to always put things to the side where there are more x's to start with (in this case the 9x side). It is really up to you

The keys to really remember is that, unless there are parenthesis involved, you should always deal with adding and subtracting items prior to multiplying or dividing "addsub before multdiv". Here are some additional examples:

Example 6:

$$4+9x-2=7x+14-8x$$
$$2+9x=-x+14$$
$$2+10x=14$$
$$10x=12$$
$$x=\frac{6}{5}$$

Example 7:

$$(4x+5)/2=7x+1$$
$$4x+5=(7x+1)\cdot 2$$
$$4x+5=14x+2$$
$$5=10x+2$$
$$3=10x$$
$$\frac{3}{10}=x$$
$$x=\frac{3}{10}$$

******* Note that in example 7 I dealt with getting rid of the division first. When there are parenthesis, you must get rid of what is outside the parenthesis first wither by distributing through or canceling it to the other side. You may have also noticed that I switched the final answer around to that the x is on the left. Finally notice that I did I step at a time only. You are expected to show one step for moving the x's and another one for moving the numbers.

Example 8:

$$5(x+3)+2x=9x+1-13$$
$$5x+15+2x=9x-12$$
$$7x+15=9x-12$$
$$-2x+15=-12$$
$$-2x=-27$$
$$x=-27/-2$$
$$x=\frac{27}{2}\ or\ 13\frac{1}{2}\ or\ 13.5$$

a-x=b

This is a case I mentioned in the last packet. It is frequently handled incorrectly so be careful. It is a good idea to remember the definition of subtraction before tackling this problem

$$a-x=b \Leftrightarrow a+-x=b$$

So say you are given the problem of

$$5-x=3$$

You may not have to actually rewrite the problem but you should think of it as

$$5+-x=3$$

With this idea in place the first step is just like normal – subtract 5 form each side

$$5+-x-5=3-5 \quad \text{or}$$

-x = -2
The negative in front of x can be cancelled out by multiplying by –1 or dividing by by –1 (they are really the same thing!)

x = 2

Here is another example without the commentary

$$29 - x = 34$$
$$29 + -x = 34$$
$$-x = 5$$
$$x = -5$$

It is a common mistake to subtract the 29 and then just forget about the "-" but hopefully you will avoid this. It is a really annoying way to loose points.

Check your answers

If you have not been checking your answers thus far, this is a really good place to start. Simply take whatever you found to be the answer and put it in for x. For example, in the last problem x = -5 so put –5 in for x

29 – x = 34
29- -5 = 34
29 + 5 = 34
34 = 34 Yes, we were right!

Checking your answer is a simple quick process which will catch a lot of errors. You do not even have to write it down. It is usually all numbers so you can often just do it in you head. Now I realize that on a test you may be pressed for time but for simple problems like this you will usually be better served by catching you answers than making sure you get to all the problems if that means leaving errors. Of course if you want to just get through the test, go back when you finish and check and recheck your answers. It is actually a valid argument to point out that if you do your check right away you may make the same mistake again (such as saying 12 + 14 = 36) If you come back a few minutes later you may catch this mistake,

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Problems solve each problem showing all your work
If there is more than one variable, solve for x

P 5-1 4x + 8 = 6
P 5-2 -3x – 9 = 2
P 5-3 12x + 9x – 2 = 8 – 1
P 5-4 13x+ 15x = 13
P 5-5 5a + 4 = 24
P 5-6 50x + 20(3x+14) = 29
P 5-7 $\frac{4x+1}{9} = 5$

P 5-8 5(x – 1) = 9
P 5-9 5 – 2x = 10
P 5-10 12 + 6x = 4x – 1
P 5-11 20 + 5x – 2x + 3x + 9x = 3 – 1 + 7x – 2x
P 5-12 3x + 6 = a
P 5-13 9x - a = 5
P 5-14 8 –x = 14
P 5-15 20x + 13 x – 52x = 48
P 5-16 4x + 9 = 3x – 1
P 5-17 16(3+2(x+5)) – 4 = 6x
P 5-18 $\frac{5x-8}{3}+12=20$ (hint if you think of it as [] +12=20 what do you do first?
P 5-19 $\frac{5x+1}{2}=\frac{9x+3}{4}$ (hint find the least common denominator and multiply both sides by it)
P 5-20 10x + 1 = 21 might as well have an easy one again

Unit 6: Decimals and graphs

Decimals and Fractions

It happens that occasionally in solving a problem that you end up with something like this

$$x = \frac{.12}{.7}$$

This is absolutely not an acceptable answer. A basic rule to always remember is that you never leave both fractions and decimals in your answer. Of course many of you reach for your calculators but you should have learned my feeling about those by now but lets say you did anyway. You would get .171428571429… There is no exact decimal answer for this and unless I specifically said to, you are not supposed to round your answers off. So how do you get an exact answer?

The method of handling this problem is actually pretty simple. Look at the problem. How many decimal places does it have at most? 2 So that means to multiply top and bottom by 100. After that we can reduce if possible.

$$\frac{.12}{.7} = \frac{.12 \cdot 100}{.7 \cdot 100} = \frac{12}{70} = \frac{6}{35}$$

Keep in mind that we need to multiply both top and bottom by the same thing. Now if you look at this problem you may realize that it would have been possible to multiply by something less than one hundred since we then had to reduce. In this particular case we could have multiplied by 50 but the time it takes to figure out what the best choice is probably more than it took to just use one hundred and then reduce.

Lets quickly go through another example.

$$\frac{.125}{.1042} = \frac{.125 \cdot 1000}{.1042 \cdot 1000} = \frac{1250}{1042} = \frac{625}{521}$$

Solving Equations with decimals

There is actually a way to avoid having decimals show up in fractions while solving. The answer is to avoid decimals in the first place. Of course you can not just skip these problems since, with our monetary system utilizing two decimal places, there are too many important problems you could not do. What you can do is handle the decimals first. Take this problem,

$$1.3x + 5.12 = 9.83$$

Similar to what we did last time, look for the most decimal places which in this case is two. This again means to multiply by 100 but rather than both top and bottom we do it to both sides.

$$100(1.3x + 5.12) = 100 \cdot 9.83$$

$$130x + 512 = 983$$

Now we can solve it like we did in the last packet which I leave up to you to do.

The key to this is to find the most decimals places of any part and then multiply by that. Be careful to not try to apply this rule to cases where it is impossible to do so. For example, it does not work for just an expression like 4.1x + 9.83 . If I were to multiply by 100 it would be changing the value of the expression. The only time I can ever introduce an operation is when there are two sides (in other words an equation). Trying to do it on an expression is like saying that 3 = 100 · 3 = 300 which is of course crazy.

You may also run into situations like this.

$$.4(3.1x+.9)=5.9$$

It is probably best to deal with this in one of two ways.

$$.4(3.1x+.9)=5.9$$
$$1.24x+.36=5.9$$
$$100(1.24x+.36=5.9)$$
$$124x+36=590$$

or by

$$.4(3.1x+.9)=5.9$$
$$10\cdot.4(3.1x+.9)=10\cdot5.9$$
$$4(3.1x+.9)=59$$
$$12.4x+3.6=59$$
$$10(12.4x+3.6)=10\cdot59$$
$$124x+36=590$$

My preference is for the first but either way just make sure you realize that you can not multiply both the number outside the parenthesis and the numbers inside. A simple experiment should convince you that it does not work.

Graphing One Variable Equations

Well it is time we dealt with some graphing problems but we will start with some simple problems. Equations with only one variable

But before we do lets go over some basic ideas for graphing in 2 dimensions.

1) Always clearly label the scale on you graphs
2) Label any important points – especially those used in graphing
3) The first number in an ordered pair is the x and the second is the y
4) The horizontal axis is the x-axis and the vertical is the y-axis
5) When in doubt – plot a bunch of points

Given any problem it is always possible to get a rough idea of the graph just by plotting points Take for example $y = 3x^2 + 9x – 2$ while others may know already that this is a U shape (technically called a parabola), you may have no idea. Set up what I call a T-Table

x	y

At this point just fill in some arbitrary points for x (try to choose both positive and negative and always use 0.

x	y
-4	
-1	
0	
2	
5	

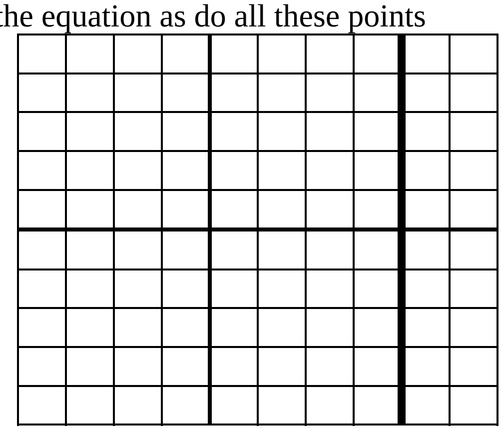
Then put each value into the equation to see what you get.

x	y
-4	10
-1	-8
0	-2
2	28
5	118

Plot the points and see what you get. You should hopefully recognize the shape as a parabola.

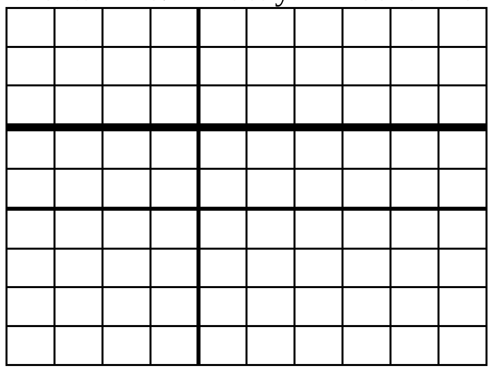
Of course with equations you recognize as lines it would only be necessary to plot two points but I am going to say that you must do three since that will provide a simple error check.

Now back to our equations with only variable. Try graphing $x = 4$ and you may think I am crazy. After all that does not appear to be the equation of any graph. It is an answer not an equation to graph right? – nope! A graph is merely the representation of all points which satisfy an equation. Look at the points (4,-3), (4,10), (4,0) and (4,3). In each case isn't it true that $x = 4$. These points satisfy the equation as do all these points



The graph of a x = any number turns out to be a vertical line. This can be hard to remember since the x axis is the horizontal one but if you just remember it is the opposite you will do okay.

Given that a vertical line is of the form x = a, what do you suppose y = 2 would look like? Well it of course is a horizontal line. In fact y = 2 looks like



Graphing Linear Equations with Two Variables – by T- Table

I have actually already talked about this actually. Since this method does not use any of the special properties of lines, all you are supposed to do is construct a T-Table using at least 3 points and plot the points. The only things I have yet to mention is that you should make sure you put little arrows on the end of the line you drew. Please note that I said line. It is supposed to be straight – ie no curves allowed. It would not be a bad idea to use a straight edge although I am not gong to require that if you can draw them to at least look straight.

P 6-1 .2x + .5 = .9
P 6-2 .12x = 4.12
P 6-3 .3x + .1x – 9.3 = 7.4
P 6-4 .1(4x + .9) = .55
P 6-5 .6x + 5 = 12
P 6-6 .1x + 13 = 52.1

Graph the following (preferably on graph paper)
(use T tables for 2 variable types and label your scale always)

P 6-7 x = 9
P 6-8 x= -2
P 6-9 y = 4.5
P 6-10 y = 26
P 6-11 x = 14
P 6-12 y = 4x + 5
P 6-13 y = 10x – 4
P 6-14 y = -3x + 1
P 6-15 y = –5x + 20
P 6-16 $y = -\frac{2}{5}x + \frac{6}{5}$

Unit 7: Graphing

Last Time we began graphing equations with one or two variables. For two variables we just used the method of plotting a number of points or in other words by T-table. This time we want to review a few shortcuts

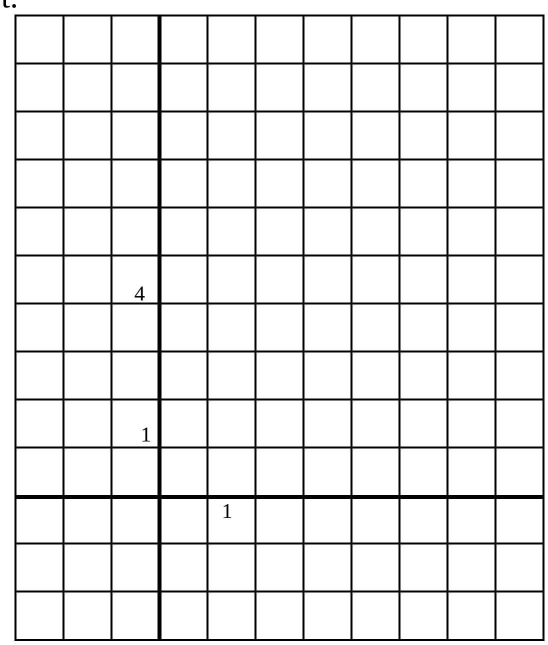
Slope – Intercept

Probably the most common way to graph lines is by using the slope and intercept. Most of the time lines are given to us in **slope-intercept** form y = mx + b. In this form the m stands for the slope of the line which as you may remember from an earlier unit, essentially tells us how fast the line is going up or down as we move from left to right. The other variable, b, tells us the y-intercept. This is the point on the y-axis, the vertical one, where the line crosses through.

Lets look at a particular example,

$$y = 3x + 4$$

This line will have 3 and we usually say that it has an intercept of 4. In reality the answer to what the intercept is would have to be (0,4). At this point you will be expected to give your answer as the coordinates of a point and not just the number. As you progress in math you will find this more and more useful. For now lets concentrate on graphing this equation. The first step is to mark the intercept.

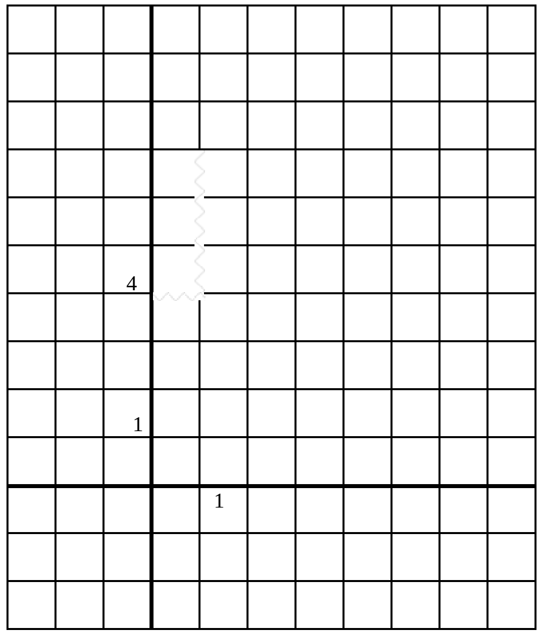


(notice that I have placed two one's on the graph to mark my scale!)

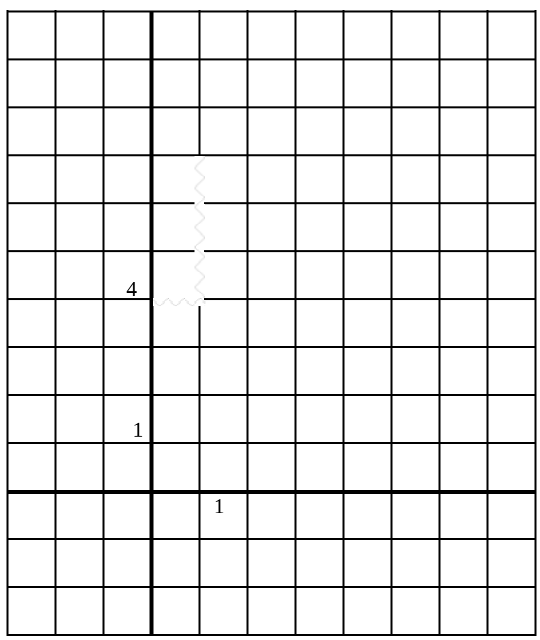
Remember that the slope was 3. No matter what the slope is we can always think of it as a fraction. In that form, the numerator (top number) represents the up or down and the bottom how far to the right we should go. In cases such as the this one we need to recall that the fractional form of 3 is $\frac{3}{1}$ We then have that we are moving only one to the right. Since the 3 is positive, we are also moving 3 up. (just as when plotting points, we use the convention that positive is up)

The key is make sure you do these moves from the intercept you already marked. It is a common error to do this from the origin but this only works if the origin was actually a solution to the equation of the line – not very common. In truth we could use any point which is on the line but by this method, the intercept is the only one we have at this time.

So from the intercept (0,4) go over one unit to the right and 3 units up before making you next point



You would not actually draw the squiggles. I included them just to help demonstrate the In the T-Table method, you were instructed to find three points as an error check but we will check this one for errors after drawing the line so now that you have two points draw the line that goes through them.



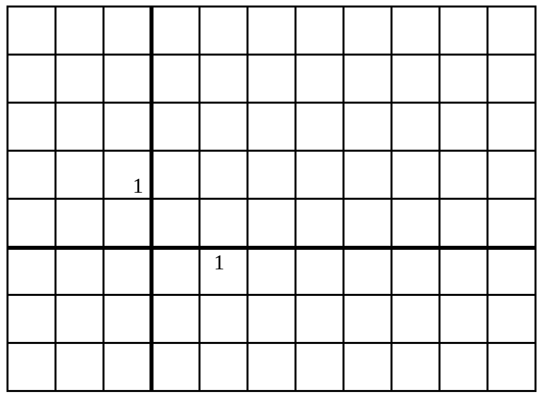
In order to check the result, plug another number into the equation for x to get an ordered pair. Verify that the point is on the line you drew. For example in this case I might plug in –1 to get the point (-1,1) which is on the line I drew.

Here is another example,

$$y = -2x + 1$$

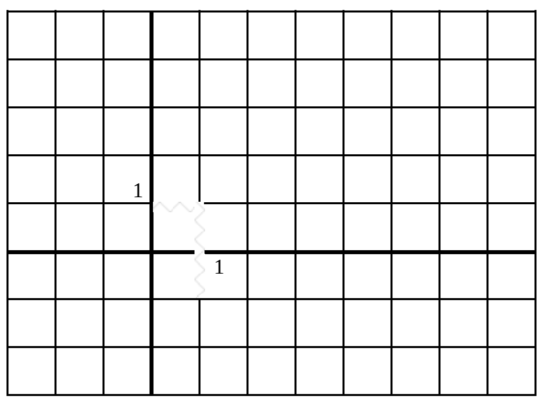
In this case the slope is –2 and the intercept is (0,1). Again the steps are as follows after the scale has been marked on the graph

1) **Draw a point at the intercept** (0,1)

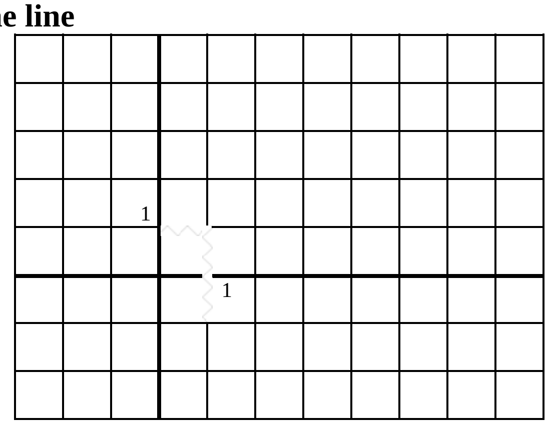


2) **Mark the slope** - In this case the slope is $\frac{-2}{1}$ so one right and 2 down

(note the negative number meant to move down but we still move right)



3) **Draw the line**



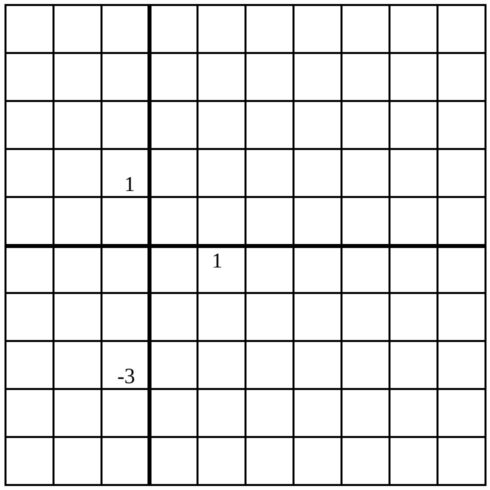
4) Check you answer

if x = -2 than the point is (-2,5) which is on my line

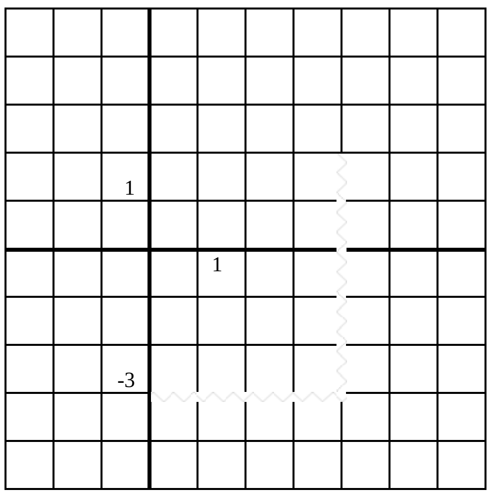
Of course not all problems will have slopes which are integers so we should look at another example. Lets try

$$y = \frac{5}{4}x - 3$$

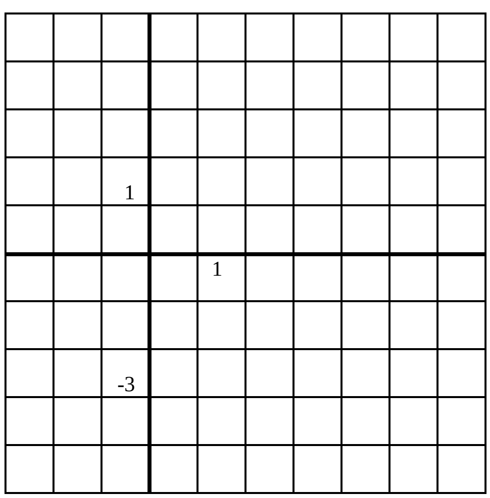
1) **Draw a point at the intercept --** (0,-3)



2) **Mark the slope** $\frac{5}{4} \rightarrow$ means over 4 to the right and up 5



3) **Draw the line**



4) Check you answer x = 1 → $\left(1, 2\frac{3}{4}\right)$ which is on the line

summary

Here are the points to make sure you remember

- The y - intercept is a point which looks like (0,4)
- We always move right the number of places in the denominator
- The top number specifies how far up or down – up if positive and down if negative
- Move the slope from the intercept – not the origin

What is the y-intercept

P 7-1 $y = 2x + 9$

P 7-2 $y = \frac{3}{2}x - 8$

P 7-3 $y = 5x + \frac{1}{2}$

P 7-4 $y = .2x + .8$

Graph the following using slope intercept - make sure you mark your scales

P 7-5 y = 4x – 2

P 7-6 $y = \frac{2}{3}x + 4$

P 7-7 $y = -\frac{1}{2}x - 2$

P 7-8 $y = -2x + 6$

P 7-9 $y = -10x + 40$

P 7-10 $y = \frac{12}{19}x + 1$

P 7-11 $y = 3x$ (Hint: think of it as $y = 3x + 0$)

P 7-12 $y = \frac{-4}{1}x + 12$

P 7-13 $y = 7x + 3$

P 7-14 $y = 2$

(hint: you did these last time but it may help to think of this as y = 0x + 2)

Unit 8: Graphing Continued : Intercepts and absolute value

Intercepts

Last time we talked about on of the intercepts – the y-intercept. We dealt with this because not only is it useful in graphing but it is also usually easy to find. Take for example the equation y = 3x+7. The number 7 is in the b position of the form y=mx+b so the intercept is (0,7). Recall that this is point at which the line would cross the y axis on a graph.

Just as we deal with where the line crosses the y-axis, we also want to deal with where the line crosses the x-axis. This point becomes very important in latter math and in physics since the x-axis is often used to represent the ground. When using this idea, the line crossing the x-axis is equivalent to the object hitting the ground. But enough with interpretations (of which there is an infinite number), the goal right now is to be able to find it for linear equations. For that, lets use the equation y = 3x+7. When writing the y-intercept we set the x value equal to 0. We do the opposite here. For the x-intercept, we have the y equal to zero.

Since the equation is rarly set up in such a format, I am not going to even bother talking about a form where the intercept can simply be read out of an equation. Rather we will use the idea of the y being 0. Take the equation and substitute 0 for the y

$$0 = 3x + 7$$

We not have a simple linear equation to solve to find x, which will turn out to be $-\frac{7}{3}$.

The x intercept is therefore $\left(-\frac{7}{3},0\right)$. Note that again it is important to remember that the intercept is again a point – a pair of values and not just a single number.

Of course usually we will want both intercepts so here are some examples where we will find both.

Example 1: **y = 8x – 20**

y – intercept

b = -20 → (0,-20)

x – intercept

y = 0 → 0 = 8x – 20

20 = 8x

5/2 = x → (5/2,0)

Answer: **y-intercept (0,-20) x-intercept (5/2,0)**

Example 2: **3x + 7y = 20**

* *note that it is not in our usual form so for both intercepts we must use the idea that the opposite variable is equal to 0*

y – intercept

x = 0 → 3(0) + 7y = 20

7y = 20

y = 20 / 7 → (0, 20/7)

x – intercept

y = 0 → 3x + 7 (0) = 20

3x = 20

x = 20 / 3 → (20/3 , 0)

Answer: y-intercept $\left(0, \frac{20}{7}\right)$ **x-intercept (**$\left(\frac{20}{3}, 0\right)$

Just as I have shown above, make sure you always write your answers as points and clearly label which intercept is which.

Graphing by intercepts

I thought I should include a little discussion about the next way to graph equations which is by intercepts. If we know both intercepts we can plot those two points and then draw the line between them. Previously I always said to do three points if plotting by points but when using the two intercepts it is acceptable to just use those two.

So for the equation 4x + 3y = 12 you would find the two intercepts

4**x + 3y = 12**

y – intercept

x = 0 → 4(0) + 3y = 12

3y = 12

y = 4 → (0, 4)

x – intercept

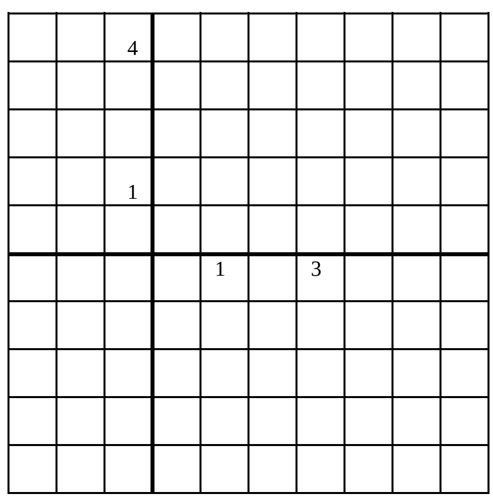
y = 0 → 4x + 3 (0) = 12

4x = 12

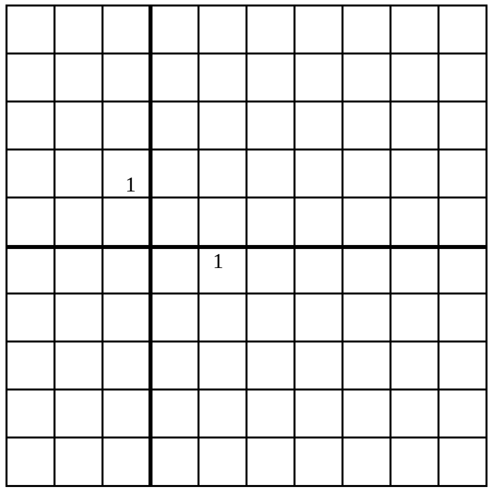
x = 3 → (3 , 0)

Answer: y-intercept (0,4) x-intercept (3,0)

Now plot the two points and draw the line through them.



Graph of Absolute Value Equations



$y = |x - 2| - 3$

Here is a kind of strange looking graphs. It is essentially a V shape. On either side of the vertex (or bounce point) the equation looks like a line but with opposite slopes on each side.

The key to graphing absolute value equation is to find the vertex. It actually is not all that hard once you understand the trick. Find the value which makes the inside of the absolute value equal to zero and use that as the x-coordinate. In this case set $x - 2 = 0$. Solve this to get $x = 2$ Therefore the vertex is (2, ?) To find the y just plug the 2 in for x and see what you get (2,-3). The next step is to make a T-Table with the vertex in the middle and two points on each side. **make sure 0 is one of the 5 points!*

Example 1:

$y = 2|2x+8| - 5$

Set $2x + 8 = 0$ and solve

$2x = -8$

$x = -4$

Vertex = (-4,?)

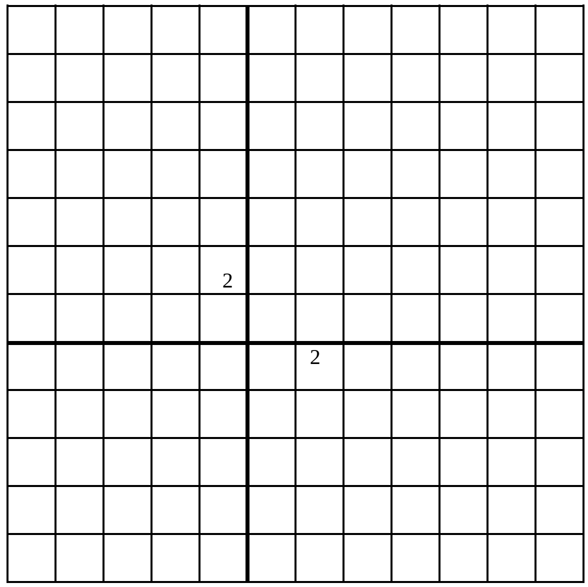
$? = 2|2 \cdot (-4) + 8| - 5$

$? = 2 \cdot 0 - 5$

$? = -5$

Vertex = (-4,-5)

x	Y
-6	3
-5	-1
-4	-5
-3	-1
0	11



Again the idea is to find the vertex by setting whatever is inside the absolute value bars equal to 0 and then making a T-Table centered around that point

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Find the x and y intercepts

P 8-1 y = 9x –4
P 8-2 y = -10x + 40
P 8-3 3x + 9y = 27
P 8-4 -7x + 2y = 15
P 8-5 3x + 7y = 12

find the intercepts and graph the following

P 8-6 y = 2x – 6
P 8-7 y = 10x + 20
P 8-8 x + 3y = 12
P 8-9 4x – 5y = 16
P 8-10 2x+ 8y = 14

find the vertex and graph the equations

P 8-11 y = | x – 6| + 1
P 8-12 y = | 5x – 10| + 7
P 8-13 y = -4| x – 1| + 1
P 8-14 y = |x|
P 8-15 y = | x + 9|
P 8-16 y = | x| + 9

Unit 9 : Finding the Equation

Now that you can hopefully graph the equation when it is given to you, we should look at the other side of it – Where did the equation come from. Specifically we are going to look at finding the equation of the line when we are given different amount of information. We will start with the easiest way and slowly go back one step of the process.

Since we generally prefer to use the slope/intercept form of the line equation (y = mx + b) , we are looking to fill in the variables m and b whenever we are writing the equation. Please note that we leave y and x in the equation of the line. With the exception of horizontal and vertical lines, we need to have both y and x in our answer. m and b on the other hand are the parameters which determine which line we have. Our goal then, no matter what information we start with is to find the slope (m) and the y-intercept (0,b).

Type 1 - Starting with slope and intercept

This is about as simple as we can make the problem. Imagine that you are told to find the equation of line when told that the slope is 4 and the y-intercept is (0,5). The answer is very simple. In effect I just told you that m = 4 and b = 5 so the equation is y = 4x + 5. That is all there is to the problem so I will just give you one more example before going on to type 2 problems.

Problem: slope = $\frac{2}{3}$ y intecept = (0,12)

Answer: $y = \frac{2}{3}x + 12$

Type 2 – Starting with slope and one point

This is going to be just a little more difficult. A typical problem would say find the equation of the line with slope equal to 3 going through (5,12). We know that m = 3 so we already have the equation $y = 3x + b$. We now need to find b. In order to do this we substitute the point (5,12) into the equation for x and y and then solve for b.

$$12 = 3 \cdot 5 + b$$

$$12 = 15 + b$$

$$-3 = b$$

We are now basically back at a type 1 problem where we know both slope and y iintercept. All that is left is to write our answer,

$$y = 3x - 3$$

Here is another example

Example 1: Slope = 4 and through (3,7)

m =4

y = 4x +b

7 = 4 · 3 + b

7 = 12 + b

-5 = b *(now at a Type 1)*

Answer: y = 4x – 5

Example 2: Slope = $\frac{3}{5}$ and through (2,9)

$$m = \frac{3}{5}$$

$$y = \frac{3}{5}x + b$$

$$9 = \frac{3}{5}(2) + b$$

$$9 = \frac{6}{5} + b$$

$$9 - \frac{6}{5} = b$$

$$\frac{45}{5} - \frac{6}{5} = b$$

$$b = \frac{39}{5}$$

answer = $y = \frac{3}{5}x + \frac{39}{5}$

This problem shows you not only how to do the problem but also how much I expect to shown when you are solving it. You need to show each of the steps shown above. Of course when there are no fractions it might take less steps such as in the following..

Example 3 slope = -4 and through (3,-8)

$$m = -4$$

$$y = -4x + b$$

$$-8 = -4(3) + b$$

$$-8 = -12 + b$$

$$4 = b$$

answer: y = -4x+4

Type 3 - given two points

This is generally the most difficult of the problems. Our goal will be to turn this type 3 into a type 2 and then into a type 1.

Example 4: through points (2,8) and (5,14)

Type 3 → Type 2

Our first step will be to find the slope.

$$m = slope = \frac{y_1 - y_2}{x_1 - x_2}$$

*** remember that it does not matter which point goes first as long as x_1 and y_1 are from the same point.*

$$m = \frac{14-8}{5-2} = \frac{6}{3} = 2$$

Pick either one of the points to go on to step 2 – it will not matter which since the correct line should have either point

Type 2 → Type 1 find b given m = 2 and through (2,8)

$$y = 2x + b$$
$$8 = 2 \cdot 2 + b$$
$$8 = 4 + b$$
$$4 = b$$

Type 1 → answer currently know m = 2 and b = 4

Answer: y = 2x+4

Example 5: through the points (3,8) and (7,1)

Type 3 → Type 2 given (3,8) and (7,1)

$$m = \frac{8-1}{3-7} = \frac{7}{-4} = -\frac{7}{4}$$

Type 2 → Type 1 given (3,8) and m = $-\frac{7}{4}$

$$y = -\frac{7}{4}x + b$$
$$8 = -\frac{7}{4}(3) + b$$
$$8 = -\frac{21}{4} + b$$
$$\frac{32}{4} + \frac{21}{4} = b$$
$$b = \frac{53}{4}$$

Type 1 → Answer given $m = -\frac{7}{4}$ and $b = \frac{53}{4}$

Answer : $y = -\frac{7}{4}x + \frac{53}{4}$

Find the equation of the line

P 9-1 slope = 5 y-intercept = (0,3)

P 9-2 slope = -3 y-intercept (0,9)

P 9-3 slope = $\frac{4}{3}$ y-intercept (0,2)

P 9-4 slope = $-\frac{3}{2}$ y – intercept $\left(0,\frac{4}{3}\right)$

P 9-5 slope = 6 and through (4,1)

P 9-6 slope = -8 and through (7,-3)

P 9-7 slope = -14 and through (3,-6)

P 9-8 slope = $\frac{1}{3}$ and through (8,5)

P 9-10 slope = $-\frac{2}{3}$ and through (-2,12)

P 9-11 through points (4,1) and (9,5)

P 9-12 through points (-5,3) and (11,12)

P 9-13 through points (14,1) and (6,3)

P 9-14 through points (4,2) and (0,7)

P 9-15 through points (8,13) and (0,0)

P 9-16 through points $\left(\frac{4}{5},8\right)$ *and* $\left(\frac{3}{2},12\right)$

P 9-17 through points $\left(\frac{1}{2},\frac{1}{3}\right)$ *and* $\left(\frac{4}{5},\frac{7}{4}\right)$

Unit 10: Solving Inequalities

Well we already dealt with how to solve equations earlier. We had a number of steps involved at times but they basically all dealt with the idea of getting the variable by itself by a series of operations which kept the equation in balance. We assumed it started out balanced and then doing things like adding the same amount to each side we could be sure it stayed balanced. Now we need to deal with situations which are obviously unbalanced from the start. What I am referring to is (as if the title of this section did not give it away) inequalities. Statements of the form *expression inequality expression* such as $3x < 7x + 2$. Keep in mind, the equations only refers to statements which use the equal sign. In truth we do almost the exact same thing with these inequalities as we did with equations. There are just a few simple yet rather important differences

Interpretation

When we were told to solve the equation $4x + 2 = 10$ we were basically being told to find the one value of x which would make the statement true. The value we found, in this case 2) could be put in the place of x to give us a true statement. $4 \cdot 2 + 2 = 10$. Which a quick calculation will show is correct.

With inequalities we do not find a single answer. We end up with things like $x < 5$ or $x \geq 8$. These answers specify an infinite number of answers. The key is that this must be the same set of numbers that the original expression referred to. Take the expression,

$$4x + 5 < 12$$

Well a quick calculation should tell you that if x equaled 2 than the left side would be 13 – a little too large but 1 gives us 9 which works. Lets say the answer is $x < 2$. This seems correct – after all 2 made the left side too big but 1 worked no problems right? Lets be sure.

0 is less than 2 so lets try that $4 \cdot 0 + 5 <? 12 \rightarrow 5 < 12$ Correct

$-3 < 6$ so try that $4 \cdot (-3) + 5 <? 12 \rightarrow -7 < 12$ correct again

Try something on the other side

$6 < 2$ is wrong and $4 \cdot 6 + 5 <? 12 \rightarrow 29 < 12$ which is also wrong

In fact I could fill up a million pages showing you examples which showed that every value which does not satisfy $x < 2$ also does not satisfy $4x+5<12$ and exmples where $x <2$ and $4x+5<12$. Unfortunately I would have to avoid examples like 1.9

$1.9 < 2$ but $4 \cdot 1.9 + 5 <? 12 \rightarrow 12.6 < 12$ Wrong!!!!!!

Remember that I said that the answer for an inequality must point to the exact same set of numbers as the original did. That means that every number that works for your answer must work in the original expression. It actually works both ways. If I tried saying the answer was $x < 1$ to avoid the 1.9 problem I would ignoring that fact that while $1.1 < 1$ is not true $4 \cdot 1.1 + 5 < 12$ is true.

What you find as the answer to the question of solving an inequality must represent the exact same set of numbers as the original question. The answer for this example must therefore be $x < \frac{7}{4}$ or if you prefer $x < 1.75$.

Solving inequalities

Enough technical stuff, how do you solve them? As I said earlier, the method is basically the exact same. There is only one additional rule. We need to deal with cases rising from the fact that while 3 < 5 it is also true that -3 > -5. If we multiply both sides of the equation by a negative the inequality in the middle must change directions.That is basically the new rule once we include the fact that we learned that division was just a form of multiplication. Solve the inequalities just like equations but whenever you multiply or divide by a negative number than you need to change the direction on the inequality sign.

Example 1

$$\begin{aligned} 4x + 5 &< 12 \\ -5 \quad &\quad -5 \\ 4x \quad &< 7 \\ \div 4 \quad &\quad \div 4 \\ x &< 7/4 \end{aligned}$$

Example 2

$$\begin{aligned} -3x + 1 &< 13 \\ -1 \quad &\quad -1 \\ -3x \quad &< 12 \\ \div -3 \quad &\quad \div -3 \qquad \text{*dividing by a negative} \\ x &> -4 \end{aligned}$$

Note that the answer included a –4 is not relevant. The only important things is that I divided by a negative so I had to change the sign around.

One final trick to be aware of is that it is proper form to always state your answer with the variable on the left so if you end up with 5 < x switch everything around to make it x > 5. It is okay to just reverse the statement as long as you remember to switch the inequality around as well.

P 10-1 $4x + 5 < 1$

P 10-2 $7x - 2 > 19$

P 10-3 $12x + 2 < 6x - 1$

P 10-4 $5x < 28$

P 10-5 $\frac{4}{3}x + 2 > 5$

P 10-6 $6x - 2 < 5x + 9$

P 10-7 $-2x + 12 > 16$

P 10-8 $5x - 10 > 3x + 9$

P 10-9 $\frac{4x+1}{3} < 5$

P 10-10 $12x + 20 > 6$

P 10-11 $-5x + 1 < 7x + 3$

P 10-12 $-\frac{2}{3}x + 1 < \frac{4}{9}$

P 10-13 $\frac{5}{6}x + 9 \le \frac{3}{4}x - 10$

Unit 11: More Inequalities & Absolute Value

Inequalities

Last time we deal with solving simple inequalities problems such as $4x + 5 < 12$. What we learned was that solving them is really no different than solving normal equations with the exception of having to switch the direction of the sign when multiplying or dividing by a negative. The real difference lay in the interpretation of the answer. Normal equations give either 1 answer or perhaps a few different answers. Inequalities of the last packet gave an infinite number of answers since any number which was either less than or greater than a certain number was the answer. This time we want to continue having an infinite number of solutions but we want to limit it a little more. By using a compound inequality we can specify a range of answers.

Compound inequalities are really a combination of two inequalities usually joined at this level by the words "and" or "or" although sometimes the symbols $\cap$ and $\cup$ are used in their place. Before we get into solving to get answers lets this lets look at how to interpret these answers. Lets suppose the answer is "$x > 5$ and $x < 10$". Lets use the following chart to check some numbers

X	x>5	x<10	x>5 and x<10
-2	No	Yes	No
0	No	Yes	No
5	No	Yes	No
6	Yes	Yes	**Yes**
8	Yes	Yes	**Yes**
10	Yes	No	No
12	Yes	No	No

Notice that when joined by the word "and" the only time the compound inequality works is when both parts were true. This idea of a set of numbers between two parts can be better seen if I rewrite the inequality as a single joined inequality.

$$x>5 \text{ and } x < 10 \rightarrow 5<x<10$$

which we can be read as either "5 is less than x less than 10" or simply as "x is between 5 and 10 non-iclusive. The noninclusive part is because both parts used strictly less than or greater than signs and not $\leq$ or $\geq$ which would include the 5 and 10 as valid answer.

One of the other tools we can use to help solve these types of problems is numbers lines. Lets look at the two parts individually and then the final answer on the number line

$x > 5$

$x < 10$

$x > 5$ and $x < 10$

Notice that valid area is just where the two individual parts overlapped.

Lets use number lines to examine this statement

x > 5 and x > 10

x > 5

x > 10

x > 5 and x > 10

This time notice that the final answer was the exact same as one of the individual parts so we can actually say that x > 5 and x > 10 ➔ x > 10 If you think about it makes sense because any number greater than 10 is automatically greater than 5 so that part of the statement is really unnecessary.

Now lets do one more

x<5 and x > 10

x < 5

x > 10

x < 5 and x > 10

I did not leave the final answers blank by accident. The term and means that the number satisfies both the first statement and the second but can you think of a single number greater than 10 which is also less than 5. It is not possible so we say the actual answer is the empty set which can also be written as {} or ∅. My preference is for the zero with a line through it since was are not dealing with other parts of set notation yet.

Now lets look at "or" or more specifically x > 5 or x<10

x	x>5	x<10	x>5 or x<10
-2	No	Yes	**Yes**
0	No	Yes	**Yes**
5	No	Yes	**Yes**
6	Yes	Yes	**Yes**
8	Yes	Yes	**Yes**
10	Yes	No	**Yes**
12	Yes	No	**Yes**

This statements actually includes everything since any number which is not greater than 5 must be less than 10 and the term “or” means that only one of the two inequalities must be true. It is the true if the first statement or the second is true. In this case this gave us the following result

$$x > 5 \text{ or } x < 10 \rightarrow \text{all real numbers or } \Re$$

**please note that if you want to just the write the letter $\Re$ instead of the phrase “all real numbers” that it must be a cursive $\Re$ since a block letter R means something else in math.

If this case of all real numbers were typical it would be rather pointless to use the term “or” but there is another type of case where it is necessary

$$x < 5 \text{ or } x > 10$$

$$x < 5$$

$$x > 10$$

$$x < 5 \text{ or } x > 10$$

Notice that the final answer is just the combination of both individual parts and not the overlap as it was with the term “and”. There really is no way to rewrite this inequality as anything else but there is for the next one

$$x > 5 \text{ or } x > 10$$

$$x>5$$

$$x > 10$$

$$x > 5 \text{ or } x > 10$$

This time it appears that the final answer is just $x < 10$. When we joined these two inequalities with the term “and” we just used the $x > 10$ since it was the more restrictive of the two but with or we use the least restrictive one $x > 5$.

Between inequalities

Lets go back to the ones which were written like $2<x<8$ or x is between 2 and 8. We can actually be given problems to be solved that look like this. For example

$$4 < 3x-2 < 7$$

If we were given just $3x-2<7$ you would hopefully know what to do. You would add two to both sides and then divide by three. If you were given $4 < 3x - 2$ you would also add two to both sides and divide by three. Since both parts are the same – do to all three parts at once

$$4 < 3x - 2 < 7$$
$$+2 \quad +2 \quad +2$$
$$6 < 3x < 9$$
$$\div 3 \quad \div 3 \quad \div 3$$
$$2 < x < 3$$

Heres another example

$$6 < 4x - 10 < 15$$
$$16 < 4x < 25$$
$$4 < x < \frac{25}{4}$$

Here is another one with a slight twist

$$7 < -3x + 5 < 12$$
$$2 < -3x < 7$$
$$-\frac{2}{3} > x > -\frac{7}{3}$$

Since I divided by a negative, I had to switch the signs around but I can not leave it this way. It is general practice to always have smaller numbers on the right so we switch this around to become

$$-\frac{7}{3} < x < -\frac{2}{3}$$

Please note whenever you switch around an inequality that you must also switch the signs around so that the open side is to the same thing it was before.

Before I forget to mention it, the same rules apply when you have $\leq$ and $\geq$ but when you graph it on a number line you use closed circles instead of open ones to indicate that the value is a part of the solution.

Absolute Value and inequalities

We showed awhile ago that normal inequalities such as |4x+2| = 6 have two solutions but what if we change it to be | 4x+2 | > 6. As is usual with inequalities this yields an infinite number of solutions but we also have to deal with the two cases absolute value brings into the mix. The best choice is to split the cases and temporarily ignor the inequality. For now just solve the two equations 4x+2 = 6 and 4x + 2 = -6. we get the results 1 and –2
Place them on a number line and we will use the method of testing values to find the proper answer.

The basic idea is to take numbers in each of the three regions to see if they work. In any region if on number works than they all do and if one number does not work than none of them will (assuming I found the correct critical points of 1 and –2.) Lets start on the left and choose –3

$$| 4 \cdot -3+2 | > 6$$
$$| -10 | > 6$$
$$10 > 6$$

That part works so I will mark that on the number line and move on to the next part and try 0

$$| 4 \cdot 0+2 | > 6$$
$$| 2 | > 6$$
$$2 > 6 \quad \text{no}$$

We will therefore leave that part blank

Lets now try the third region – lets pick positive 2

$$| 4 \cdot 2+2 | > 6$$
$$| 10 | > 6$$
$$10 > 6$$

This works again so now I have my final answer.

If you have looked over the previous compound inequalities you may recognize this as an "or" case. Therefore $| 4x +2| > 6 \rightarrow x < -2$ or $x > 1$

What if it was $| 4x +2 | < 6$. We would then have gotten the following number line

We would then have $| 4x +2| < 6 \rightarrow x > -2$ and $x < 1$ which can be rewritten as $-2<x<1$

The key to solving these absolute value inequalties is to start treating them like there was no inequality. Get the absolute value part by itself and then split into two cases. At this point rather than calling the two numbers you get the answers you call them critical points and use them to divide the number line into regions which you can then test. Here is one more to demonstrate.

$$5|4x+1|+3<23$$
$$5|4x+1|<20$$
$$|4x+1|<4$$

$4x + 1= 4$	$4x + 1 = -4$
$4x =3$	$4x=-5$
$x = 4/3$	$x = -5/4$

-3	0	2
$5\| 4 \cdot -3+1\| + 3 < 23$	$5\| 4 \cdot 0+1\| + 3 < 23$	$5\| 4 \cdot 2+1\| + 3 < 23$
$58 < 23$ no	$8 < 23$ yes	$48 < 23$ no

Therefore

So the answer is $-5/4 < x$ and $x < 4/3$ in other words $-5/4 < x < 4/3$

Draw a number line to represent each of the following and if possible write a single inequality to represent the same set

P 11-1 $x < 2$ and $x < 9$
P 11-2 $x > 5$ or $x < -2$
P 11-3 $4 < x$ and $x < 9$
P 11-4 $12 > x$ and $x > 8$
P 11-5 $x < 9$ or $x < 3$
P 11-6 $x < -5$ or $x > 0$

Solve – give both a number line and an inequality answer

P 11-7 $-40 < 5x < 60$
P 11-8 $3 < 4x + 1 < 10$
P 11-9 $5 < -6x + 24 < 15$
P 11-10 $| 2x + 1 | < 9$
P 11-11 $| 5x + 1 | > -2$
P 11-12 $4 + | 3x | > 6$
P 11-13 $5 |6x – 1| + 1 > 25$
P 11-14 $3|2x+6| < 26$
P 11-15 $8|4x – 2| + 10 > 4$

Unit 12: Solving Systems of Equations part 1

We have already dealt with solving equations such as $3x + 2 = 17$ where the goal was to find the value of x which is the solution of the equation. By saying the value of x is 5 we are saying that if I place 5 in for x that the equation would be true. ($3*5+2=17 \rightarrow 17=17$). We are going to start taking this idea one step further. Imagine starting with an equation like $2x + 3y = 18$. A solution to this would have to consist of two different values – one for x and one for y. In this case (x,y) could be (3,4) because $2 \cdot 3 + 3 \cdot 4 = 18$. Of course it is also true to say that (0,6) or (9,0) or (6,2) or (4,10/3) … In fact the equation is a line so there is an infinite number of possibilities. Now lets use another equation $y = 5x – 28$. This also would also have an infinite number of solutions but only even with all those, only one is the same as the list for the first equation. The point (6,2) works in both equations so it is referred to as the solution of the system. When we are finding the solution to a system of equations, we are looking for all points which are part of both equations. If you were to graph the equations, it would be the point(s) where the two graphs cross.

So how many solutions are there? For now we will just be dealing with lines which can only cross in three different ways.

one solution -- the lines cross in one place

zero solutions -- the lines were parallel and so do not cross

infinite solutions – the two equations turned out to be the same line

We will only deal with the most common case – one solution.

There are a number of ways to find the point one of which is substitution. The basic idea is that if two things are equal than they should be interchangeable. Lets use the two equations mentioned above to demonstrate

$$2x + 3y = 18$$

$$y = 5x – 28$$

While for virtually all the points on the line represented by $2x+3y = 18$ it is not true to say that $y=5x-28$, it is true for the one point we want. Given then that y is the same as $5x-28$, we should be able to replace any occurrence of y with $5x-28$. Our next step is to make that substitution in the first equation.

$$2x + 3y = 18 \rightarrow 2x+3(5x-28) = 18$$

Since there is now only one variable in the equation we should be able to solve it

$$2x + 3(5x-28) = 18$$

$$2x + 15x - 84 = 18$$

$$17x - 84 = 18$$

$$17x = 102$$

$$x = 102/17 = 6$$

We now have part of our solution. We know that x is six but we still need to find y but that just requires a substitution of this 6 for x in either of the two original equations.

$$y = 5x - 28$$

$$y = 5(6) - 28$$

$$y = 2$$

Our final answer is then (6,2).

Here is another example:

$$5x - 3y = -19$$

$$x = 4y + 3$$

$$5(4y+3) - 3y = -19$$

$$20y + 15 - 3y = -19$$

$$17y + 15 = -19$$

$$17y = -34$$

$$y = -2$$

$$x = 4(-2) + 3 = -8+3 = -5$$

Answer: (-5,-2)

Both these examples we easily set up for solving by substitution. It is also possible to solve other type of equations.

$$2x + 4y = 2$$

$$5x - y = 16$$

If you are given this, you need to take one of the two equations and solve it for one of the variables. The best choice here is the second equations.

$$5x - y = 16$$

$$-y = -5x + 16$$

$$y = 5x - 16$$

With this we can now do the substitution into $2x + 4y = 2$

$$2x + 4(5x-16) = 2$$

$$2x + 20x - 64 = 2$$

$$22x - 64 = 2$$

$$22x = 66$$

$$x = 66/22 = 3$$

Now put that back into either one of the original problems. (you could use your solved for version but by using one of the originals it helps perform an error check)

$$5x - y = 16 \rightarrow 5(3) - y = 16$$

$$15 - y = 16$$

$$-y = 1$$

$$y = -1$$

Answer: (3,-1)

Solve by substitution

P 12-1 $3x + 5y = 57$ $y = 2x + 1$
P 12-2 $2x - y = 6$ $y = -2x + 6$
P 12-3 $4x + 9y = -3$ $y = 4x + 13$
P 12-4 $2x + 3y = 14$ $y = 5x - 12$
P 12-5 $4x - y = 10$ $y = 2x + 1$
P 12-6 $2x + 5y = 7$ $3x - y = 2$
P 12-7 $3x - 2y = 2$ $x + 4y = 10$
P 12-8 $5x + 3y = 3$ $6x + y = 14$
P 12-9 $9x + 2y = 25$ $x = -2y + 5$
P 12-10 $4x + 2y = 2$ $5x - 10y = 15$
P 12-11 $10x + 15y = -3$ $5x - 3y = 14$

Unit 13: More on Solving Systems of Equations

Just to remind you, solving a system of equations means that we are looking for the points (although usually only one) which satisfies a series of equations. Last time we just dealt with two equations and a method known as substitution. The priniciple idea is that we can take two equal expressions and replace one with the other. This allowed us take the following system

$$3x + 5y = 12$$
$$y = 2x + 1$$

and solve it by using the fact that, at the point we are interested in, y and 2x + 1 are the same thing. This being true we state that instead of 3x + 5y = 12 that 3x + 5(2x+1) = 12. After all, y and 2x + 1 are the same so why shouldn't 5 times y be just like 5 times 2x+1? Unfortunately we discovered that while this worked rather well with systems like that above, it was not so easy to use if we are given

$$3x – y = 7$$
$$5x + y = 17$$

In order to solve this with substitution, we had to solve one of the equations for one of the variables and then use substitution. We are going to use the same principle behind that method but in a new way called linear combination.

Linear combination actually operates on two principles. The first is like that above where we should be able to substitute one thing with something that it equals. The second idea is that you can add to both sides of an equation without changing the equation as long as you add the same thing to both sides. Allow me to demonstrate. Lets add 17 to both sides of the first equation.

$$3x - y = 7$$
$$3x – y + 17 = 7 + 17$$
$$3x – y + 17 = 24$$

Now that did not seem terribly useful but lets remember the second equation which said that 5x + y = 17. With that in mind we can use our second principle, substitution.

$$3x – y + 17 = 24$$
$$3x – y + (5x + y) = 24$$

A little algebra shows that the –y and + y cancel out leaving just 8x = 24 or x =3.

Of course that that is the whole point of linear combination – to have one of the two variables cancel out. Of course we usually don't write it out like this. This just demonstrated why this method worked.

Here is a more typical problem:

Example 1: $4x - 3y = 19$

$$\begin{array}{r} 4x - 3y = 19 \\ + \underline{\ 5x + 3y = 17} \\ 9x \quad = 36 \end{array}$$ after adding the y's cancelled

$$x = 36/9 = 4$$

Now that we have x, put it into either equation to find y

$$5(4) + 3y = 17$$

$$20 + 3y = 17$$

$$3y = -3$$

$$y = -1$$

Answer: (4,1)

Of course if we had used the other equation we would still have gotten –1. If you try it and get something different than the x value had to be wrong.

Of course it is not always y that will cancel out.

Example 2:

$$\begin{array}{r} 5x - 8y = -7 \\ \underline{-5x + 10y = 15} \end{array}$$

add them together $2y = 8 \rightarrow y = 8/2 = 4$

Substitute back in $5x - 8(4) = -7$

$$5x - 32 = -7$$

$$5x = 25$$

$$x = 5$$

Answer: (5,4)

Both these problems had something in common – simple adding cancelled out a variable but what if we are given something like this

Example 3:

$$5x + 6y = 16$$

$$6x + 3y = 15$$

Adding this time gives us 11x + 9y = 31 which we still can not solve. Here is the trick to this type of problem. In the first equation there is a +6y. To cancel that out we would need the other equation to have a –6y. It does not have this but we can make it have this by multiplying the entire equation by a negative two. Remember that not only can we add the same thing to each side but we can also multiply by the same thing on each side

$$5x + 6y = 16 \rightarrow \quad 5x + 6y = 16$$

$$(-2)(6x + 3y = 15) \rightarrow \underline{-12x - 6y = -30}$$

Now we can add with something canceling out. $-7x = -14 \rightarrow x = 2$

Substituting back in we will find that y = 1 so the answer is (2,1).

Here is another one:

Example 4: $10x + 3y = 36$

$30x - 24y = 42$

To cancel out the 30x in the second equation we need a –30x so multiply the first equation by –3.

$$-3(10x + 3y = 36) \rightarrow -30x - 9y = -108$$

$$30x - 24y = 42$$

$$-33y = -66 \rightarrow y = 2$$

Substitution of this back into either equation gives us x = 3 so the answer is (3,2).

You may have noticed that multiplying the first equation by eight would also have worked since in that event the y's would have cancelled out.

Solve each system by linear combination – be sure to show your work including the substitution that takes place after you find the first value

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P 13-1 $4x + 5y = 17$

$3x - 5y = 4$

P 13-2 $-2x + 8y = 26$

$2x - 9y = -30$

P 13-3 $5x + y = 12$

$8x - y = 1$

P 13-4 $6x + 2y = 14$

$-6x + 3y = 5$

P 13-5 $4x + 8y = -4$

$2x - 5y = 7$

P 13-6 $20x + 18y = 4$

$13x - 9y = 1$

P 13-7 $-4x + 5y = -32$

$7x + 10y = 131$

P 13-8 $10x + 3y = 9$

$11x + 12y = 8$

P 13-9 $24x + 9y = 3$

$13x - 36y = 49$

Unit 14: Solving Even More Systems

The last two sections already dealt with the two primary methods of solving systems of equations – substitution and linear combination. I think most of you will admit that, although it was a little strange at first, even linear combination is not really all that difficult. Of course there are a few things we need to handle yet like unusual results and two multiplications

Unusual Results

There is really two different kinds of unusual results. Here is an example of the first.

$$3x + 6y = 4$$
$$x = -2y+5$$

You would solve this system by substitution as follows

$$3(-2y+5) + 6y = 4$$
$$-6y + 15 + 6y = 4$$
$$15 = 4$$

Now that can't be write. Even a first grader knows that 15 does not equal 4 so what happens – did I make a mistake. Well actually know. Our idea of substituting –2y + 5 for x was based on the assumption that there was a point where the two lines crossed. These lines were actually parallel and therefore don't cross.

Here is another one

$$4x + 2y = 6$$
$$y = -2x + 3$$
$$4x + 2(-2x+3) = 6$$
$$4x - 4x + 6 = 6$$
$$6 = 6$$

This time the system did not lead to a incorrect statement but rather one that is kind of a no brainer. I mean of course 6 equals 6. As it turns out, unlike the last one which meant there was no solution, this one has an infinite number of solutions. In truth any point on the one line is also on the other because the two equations are actually representing the same line. That is usually how we state our answer is that they are the same line.

In summary for this part, if you end up with a statement which makes no sense like 2=1 than the answer is no solution because they are parallel lines. If you get a statement which is always true like 3=3 than the answer is infinite, or same line

Multiple Multiplications

When we dealt with linear combination before we were always either able to just add the equations as they were given or do so after multiplying one line by something. Look at the following example

$$4x + 3y = 15$$
$$6x + 5y = 23$$

Although I could find a fraction to multiply 4 by to make it a 6 (3/2), we generally avoid fractions since they can lead to too many errors. I could look at the y's but that won't work either in this case the best choice to find the a common multiple of either 4 and 6 or the 3 and 5 (it won't matter which). Well 12 is a multiple of both 4 and 6 so I am going to get one line to be a 12x and the other a –12x.

$$3(4x+3y = 15) \rightarrow 12x + 9y = 45$$
$$-2(6x+5y = 23) \rightarrow -12x - 10y = -46$$

Something will cancel now so I add to get –y = -1 or y = 1. Now that I know that y = 1 I substitute that back into one of the original equations

$$4x + 3 \cdot 1 = 15$$
$$4x + 3 = 15$$
$$4x = 12$$
$$x = 3 \text{ so the answer is } (3,1)$$

The key is to find any multiple of the two coefficients and get both lines to have that by multiplying both lines to get it. Note you do not need to worry about finding the least common multiple because I could have multiplied by 6 and –4 above and had 24 and –24x to cancel out. Here is one more example of this type before I give you the problems.

$$4x + 7y = 9$$
$$5x - 3y = 23$$

It does not really matter which variable I choose to work with but I will use the y's where a multiple of 7 and 3 is 21

$$3(4x + 7y = 9) \rightarrow 12x + 21y = 27$$

$7(5x - 3y = 23) \rightarrow 35x - 21y = 161$

$47x = 188$

$x = 188/47 = 4$

with x equal to 4 we substitute back in

$4(4) + 7y = 9$

$16 + 7y = 9$

$7y = -7$

$y = -1 \rightarrow$ answer is therefore (4,-1)

Some of these will only require multiplying one equation, some will have no solution and some will be infinite- same line

P 14-1) $5x + 4y = 50$

$3x - 3y = 3$

P 14-2 $7x + 2y = 12$

$14x - 4y = 4$

P 14-3 $5x + 6y = 14$

$10x + 12y = 28$

P 14-4 $-2x + 8y = 12$

$7x + 3y = 14$

P 14-5 $9x + 3y = 12$

$18x + 6y = 20$

P 14-6 $10x + 3y = 15$

$20x + 12y = 60$

P 14-7 $-2x - 3y = -1$

$3x + 9y = -3$

P 14-8 $5x - 6y = -4$

$4x + 10y = -18$

Unit 15: Exponents

So far this year we have dealt mostly with linear equations and systems. These can model quite a few situations but not everything. We need to be able to model a number of equations using quadratics such as $y = -9.8t^2 + 5t + 12$ representing the trajectory of an object and perhaps even high powers. Before we get into those situations we need to review some of the basic properties of exponents so we can handle the following situations:

Product of powers: $x^2 \cdot x^3$

Division of powers: $\frac{x^6}{x^2}$

Power of a power: $\left(x^4\right)^3$

Please keep in mind a few rules we learned earlier this year such as $x^0 = 1$ and $0^x = 0$ unless x = 0 since 0^0 is undefined. You may also remember that x^3 means $1 \cdot x \cdot x \cdot x$ although for most of this unit we will be ignoring the 1 since it rarely makes a difference.

Product of powers

By going back to the definition we can handle the product of powers explanation without much difficulty. Since $x^2 = x \cdot x$ and $x^3 = x \cdot x \cdot x$ we get

$$x^2 \cdot x^3 = (x \cdot x) \cdot (x \cdot x \cdot x)$$

$$x^2 \cdot x^3 = x \cdot x \cdot x \cdot x \cdot x$$

$$x^2 \cdot x^3 = x^5$$

You may already see the pattern but just in case you are not convinced lets look at another example.

$$x^4 \cdot x^5 = (x \cdot x \cdot x \cdot x) \cdot (x \cdot x \cdot x \cdot x \cdot x)$$

$$x^4 \cdot x^5 = x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$$

$$x^4 \cdot x^5 = x^9$$

Here is a summary of the results 2 and 3 → 5
4 and 5 → 9

The pattern is that when multiplying two terms, which have the same base, you add the exponents. Here is one more example

$$x^{30} \cdot x^{532} = x^{562}$$

Try writing that one out like I did the other two. It will take you awhile but the rule using 30+532=562 is pretty simple.

Division of Powers

We can again use the definition of an exponent to figure this one out Since $x^6 = x \cdot x \cdot x \cdot x \cdot x \cdot x$ and $x^2 = x \cdot x$ we get

$$\frac{x^6}{x^2} = \frac{x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x}$$

Since it is all multiplication we can cancel terms which are on top and bottom

$$\frac{x^6}{x^2} = \frac{\not{x} \cdot \not{x} \cdot x \cdot x \cdot x \cdot x}{\not{x} \cdot \not{x}}$$

$$\frac{x^6}{x^2} = \frac{x \cdot x \cdot x \cdot x}{1} = x \cdot x \cdot x \cdot x$$

$$\frac{x^6}{x^2} = x^4$$

Again before making any rash conclusions lets look at another example

$$\frac{x^8}{x^3} = \frac{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x}$$

$$\frac{x^8}{x^3} = \frac{\not{x} \cdot \not{x} \cdot \not{x} \cdot x \cdot x \cdot x \cdot x \cdot x}{\not{x} \cdot \not{x} \cdot \not{x}}$$

$$\frac{x^8}{x^3} = \frac{x \cdot x \cdot x \cdot x \cdot x}{1} = x \cdot x \cdot x \cdot x \cdot x$$

$$\frac{x^8}{x^3} = x^5$$

Here is a summary of the results 6 and 2 → 4
8 and 3 → 5

Remembering that multiplication and division are opposite operations, it is not really unexpected that since multiplication was addition that division would be subtraction.

If you want to know what $\frac{x^{12}}{x^5}$ is just do 12 – 5 to get 7 and the answer of x^7.

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Power of a Power

We need to use the definition and the product of powers to figure this one out. Lets suppose that we are given the problem

$$\left(x^4\right)^3$$

We need to realize that the third power on a complex term is just like any other so

$$\left(x^4\right)^3 = \left(x^4\right) \cdot \left(x^4\right) \cdot \left(x^4\right)$$

Now the problem looks like a product of powers where the rule was to add. So 4+4+4 = 12
So

$$\left(x^4\right)^3 = x^{12}$$

As long as we did two before saying the rule before we might as well do the same thing here.

$$\left(x^5\right)^7 = x^5 \cdot x^5 \cdot x^5 \cdot x^5 \cdot x^5 \cdot x^5 \cdot x^5$$

since 5+5+5+5+5+5+5 = 35

$$\left(x^5\right)^7 = x^{35}$$

Of course 4·3=12 and 5 · 7 = 35 which gives us our rule. If you have a term with a power to another power than multiply the exponents. This even works when there are multiple terms inside the parenthesis as long as everything inside is either multiplication or division

Example 1: $\left(x^2y^5\right)^8 = x^{2\cdot 8}y^{5\cdot 8} = x^{16}y^{40}$

Example 2:

$$\left(2x^3y^4w\right)^5 = 2^{1\cdot 5}x^{3\cdot 5}y^{4\cdot 5}w^{1\cdot 5}$$
$$= 2^5x^{15}y^{20}w^5 \quad .$$
$$= 32x^{15}y^{20}w^5$$

For the next several examples remember that negative exponents mean a reciprocal $x^{-2} = \dfrac{1}{x^2}$

Example 3: $\left(\dfrac{4x^8y}{5x^3w^3}\right)^4$ look inside the parenthesis to simplify – the x's and y's

$$\left(\frac{4x^8y}{5x^3w^3y^4}\right)^4 = \left(\frac{4x^{8-3}y^{1-4}}{5w^3}\right)^4$$
$$= \left(\frac{4x^5y^{-3}}{5w^3}\right)^4$$
$$= \left(\frac{4x^5}{5w^3y^3}\right)^4$$
$$= \frac{4^4x^{20}}{5^4w^{12}y^{12}}$$

Example 4: $\dfrac{6x^2y^7z^3}{t^5y^6z^8} \cdot \dfrac{5xyz^3}{2x^5}$

First multiply numerators and denominators

$$= \frac{30x^3y^8z^6}{2t^5y^6z^8}$$

Now see if anything can simplify

$$= \frac{15x^3y^2z^{-2}}{t^5}$$

$$= \frac{15x^3y^2}{t^5z^2}$$

Problems -- if you have a number with an exponent of 5 or less evaluate it
--- do not leave any negative exponents

P 15-1 $x^5 \cdot x^{10}$

P 15-2 $\left(2x^2\right)^3$

P 15-3 $\left(5^4x^4y^6z^{-2}\right)^7$

P 15-4 $\frac{5x^2}{x^5}$

P 15-5 $\frac{4x^3m^4p^6}{7xm^2p^2}$

P 15-6 $\frac{12xy}{z^3} \cdot \frac{3x^2}{z^4}$

P 15-7 $\left(\frac{4w^3d^2}{5x}\right)^{-3}$

P 15-8 $\left(\frac{x^8y^9}{xy^3}\right)^4$

P 15-9 $\frac{5x}{y} \cdot \frac{2x}{8y^3}$

P 15-10 $\frac{12x^2y^{-3}}{4xy^4zw^2} \cdot \frac{5y^2z^3}{w^9}$

P 15-11) $\frac{1x^2y^3}{7y^8z^9} \cdot \frac{4y^5z^6}{x^{10}}$

Square Roots

Last time we reviewed how to handle exponents in a number of situations such as the multiplication, division, power of a power, zero and even negative exponents. Of course the most common case, other than a power of one which we usually do not even write, is to have a power of two. Having a second power leads to square roots.

What is a square root? Lets suppose that x is the square root of b. In other words $x = \sqrt{b}$ This being the case, then $x^2 = b$. For example, $3 = \sqrt{9}$ because $3^2 = 9$.

When finding square roots we use the definition to puzzle it out – usually by trial and error. Now lets suppose that we want to find $\sqrt{49}$. We are actually asking what number to the second power equals 49 ie

$$[\ ]^2 = 49 \quad or \quad \_\_ \cdot \_\_ = 49$$

The second version represents the idea of "what number times itself equals 49". If you memorized your multiplication table you have probably already come up with the answer of seven. $7 \cdot 7 = 49$ so $\sqrt{49} = 7$

Example 1: Lets try a larger one $\sqrt{196}$

Try some values

$10^2 = 100$ too small
$20^2 = 400$ way too large
$12^2 = 144$ too small
$13^2 = 169$ too small but closer
$14^2 = 196$

Therefore $\sqrt{196} = 14$

Example 2: $\sqrt{2431}$

Again try some values

$100^2 = 10000$ too large
$60^2 = 3600$ too large still
$50^2 = 2500$ a little too large
$49^2 = 2401$ a little too small

Now what? We tried 50 and it was too large but 49 is too small. We could go into decimal places like trying 49.5 which when squared is 2450.25 but still a little too large. As it turns out, we could never find the exact value since just like π, $\sqrt{2431}$ is an irrational number which means that it goes on forever without repeating. We would want to round it off if that was what the directions indicated you should do but what if the directions are just to simplify.

Simplifying square roots

In most cases we are not given square root problems which turn out as exact answers. Some of these are irrationals which must be left alone or rounded off. Others can be simplified such as $\sqrt{75}$. A quick check shows that the answer is between 8 and 9 since it is between 64 (8^2) and 81(9^2). We know use this property of square roots

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

We could say that $\sqrt{75} = \sqrt{5}\sqrt{15}$ but that is not going to help us much since neither the square root of 5 nor 15 is a nice answer. Try this

$$\sqrt{75} = \sqrt{25}\sqrt{3} \qquad \text{but since } \sqrt{25} = 5$$

$$\sqrt{75} = 5\sqrt{3}$$

What we always want to look for is a pair of numbers which divide the number inside the radical where one (or both) is a perfect square. Perfect squares are integers such as 1, 4, 9, 16, 25, 36, 49, 64, … because these numbers are the squares of 1, 2, 3, 4, 5, 6, 7, 8 respectively. If a number of this type (other than 1) divides the number in the radical evenly than the radical can be simplified.

Example 3: